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Quality and exposure control in semiconductor manufacturing. Part I: Modelling

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The purpose of this paper is to present a heuristic algorithm for quality control planning from an insurance perspective. The approach proposed here is designed to judiciously allocate controls in two stages: one to manage risk exposure, in terms of potential product loss, and the second to improve the effectiveness of the controls themselves. The method employed to evaluate this algorithm is based on three comparisons. One of these is presented in the first part of the paper and the other two in the second part. The test provided in this first part of the paper is performed on a simplified case study and compares the proposed heuristic algorithm (HA) to an optimised allocation (OA) method. The main objective of this paper is to present, in detail, a quality control planning method which allocates measurement resources with the aim of remaining below a threshold limit of risk exposure and of improving the effectiveness of the controls. The evaluation presented in the second part of the paper underscores the potential, as well as the limitations, of the proposed algorithm.

Keywords: control plan; risk exposure; quality control allocation

1. Introduction

In high-mix semiconductor manufacturing lines, the growth or expansion of uncertainty about the health of processes and products often leads to major scrap events. Once a control (the set measurement, and the associated analyses and actions taken, if required) has been applied, the real production losses can be ascertained. In a worst case scenario, thousands of products are scrapped, generating a major production disruption. Even if quality controls are in place all along the production line to protect manufacturing systems from tool drift, the monitoring of risk exposure is not well defined and fairly ineffective operationally.

Our concern in this paper is controlling exposure to risk, in terms of the number of products potentially impacted. As shown in Figure 1, our focus is quality control and improving the design of the quality control plan.

The proposed HA is aimed at reducing the extent of potential losses by minimising the number of products of doubtful quality and making it easier to detect the underlying causes of undesirable events through a judicious allocation of control effort.

A classical method of generating quality control plans is to base them on risk analyses that can be performed throughout product and process life-cycles (Department of Defense 1980, Villacourt 1992, Tixier et al. 2002). Ideally, a control plan should take these analyses into account and be capable of detecting 'known failures'. In practice, however, it is impossible to ensure that 100% of potential failures have been identified through risk analysis (Mili et al. 2009). In an effort to remedy this situation, research has been conducted into continuously updating these analyses with unwanted manufacturing events (Mili et al. 2009, 2010). Another way of tackling this issue is to use available mechanisms to monitor some key parameters for deviations. Such controls are rooted in Statistical Process Control (SPC) techniques (Spanos 1991, Montgomery 2004).

In practice again, interactions between production and quality control are rarely, if ever, addressed. This difficulty is exacerbated by the fact that control plans are often designed by teams in distinct areas of manufacturing (Schippers 2001) – reliability, product engineering, process engineering, maintenance, quality, and safety, among others. Controls reveal the presence of potentially faulty products, which we refer to as material at risk (MAR) (Bean 1997). The level of MAR is directly influenced by production and quality control plans. It can also be strongly influenced by operations management, and particularly by the quantity produced between two controls.
Several researchers have investigated this connection. Hsu and Tapiero (1989) carried out pioneering work in this area by proposing a link between operations management and SPC control charts. Gershwin and Kim (2005) and Colledani and Tolio (2011) also studied the way in which quality and operations control can be linked. In particular, Colledani in his paper designs the buffer size of the control machine taking into account quality and cycle time expectations (Colledani 2008, Colledani and Tolio 2011). In every development, a balance between the cost of non-conforming products and the cost of buffers must be established. Industry also proposes adaptive techniques for production control plans (Mouli and Scott 2007), scheduling (Purdy 2007) and the technology life-cycle (Bousetta and Cross). Recent university-industry joint studies have investigated the dynamic quality control of defects caused by shop floor operations (Shanoun et al. 2011) (dealing with skipping), (Dauzere-Peres et al. 2010) (dealing with scheduling). In most cases, quality control plan adjustments are driven by capacity limitations, or by productivity or cycle time improvement campaigns.

The interaction between production and quality control is a highly intricate one. Considering the amount of \( MAR \) in manufacturing line assessment can be seen as an opportunity to complement traditional approaches in both areas, and then to introduce a means to manage them jointly.

This paper focuses on the integration of the concept of uncertainty into models of quality control plan design, under the strong influence of the production-planning viewpoint. The goal is to keep \( MAR \) below a threshold limit \( RL \), which is the insurance capacity of the production system. Scrapping a quantity of products below this limit would be manageable and tolerable, while scrapping a quantity above it would generate a major disruption in the manufacturing system.

The paper is structured in five parts: a literature review in Section 2, the model and the proposed approach in Section 3, a numerical example for illustrative purposes and a discussion of the example in Section 4, and a comparison between this method and an optimal allocation of quality controls in Section 5. The paper concludes, in Section 6, with an outline of possible future enhancements to the model.

2. Literature review

The design of a process control plan must span various disciplines, in order to incorporate the appropriate layers of protection (Schippers 2001). Accordingly, this review is structured around a number of aspects of process control activities addressed in domains other than that of semiconductors: risk management, statistical process control, inspection allocation and process control.

2.1 Quality control plan

Almost all semiconductor manufacturers are called upon to provide updated risk analyses on their processes and products, and sometimes their tools. The objective is to reassure customers of their ability to deliver quality products on time and in the required quantities. The analyses are concerned with operational risks, which have to be identified, evaluated – often using ranking techniques – and mitigated. Ideally, there will be follow-up as well. These risks take the form of extraordinary events related to operational hazards. Customers expect manufacturers to base control plans on these risk analyses.

FMECA (failure mode effects and criticality analysis) is one of the most commonly applied techniques (Department of Defense 1980, Villacourt 1992); however, there are many others. A general survey of modern methodologies for managing risk can be found in Tixier et al. (2002). In risk analysis, layers of protection are explicitly mentioned. In FMECA, they are listed in the ‘Detectability’ column. However, few of these methods link risk analyses and actual control plan strategies in a detailed and efficient way. From the risk analysis perspective, controls are often mentioned in a generic phrase, like ‘Control with SPC’ or ‘Perform maintenance’. To make them fully operational, a specialist in the field must rework these phrases. For example, ‘Perform maintenance’ should be
expanded in a maintenance information system to include descriptive data (a label, the targeted tool, the associated
spare parts, maintenance frequency, operating mode, etc.). However, this manual operation disconnects risk
analysis from the origin of the control functions. To tackle this issue, Mili et al. (2009, 2010) and Aymen and
Bassetto (2008) proposed an activity model joining risk elicitation, risk evaluation and the risk control plan, and
applied the framework to defining the maintenance priorities of a photolithography workshop. Every time a failure
is detected, a new risk analysis must be performed and the control plan revised.

Bean (1997) presents the interesting development of an in-line dynamic inspection plan based on the probability
of deviations due to measurement errors. His work also introduces the notion of $MAR$, as every product, between
two samplings, is prone to acquiring defects. Bean does not mention what kinds of failure are monitored, but rather
discusses the potential impacts on which inspection plans are based, whatever their causes. His concept of $MAR$ has
inspired other authors, like Dauzère-Pérès et al. (2010), who recently investigated the real-time measurement of
$MAR$ and its use in real time control decisions. They introduced threshold limits to $MAR$ and computed a
composite state indicator (the global sampling indicator) for the manufacturing system to enable selection of the
product that has to be controlled first. In doing so, they extended the algorithm presented in Shanoun et al. (2011)
for application on the shop floor. Products are selected and ordered for control based on their capacity to maintain
the highest margin between the limits of risk exposure and the actual state of the tool. Their algorithm is employed
to schedule activities for monitoring defective parts.

2.2 Control plan design at the SPC level

Linking process control and risks can be viewed as an effort to rationalise inspection and quality control. We focus
now on the strategies developed to design inspection policies and update them to include risk evaluations. SPC field
have been the target of efforts to design and update controls throughout a technology or production life-cycle
(Montgomery 2004). For a complete overview of the process control tools used in the semiconductor industry and
the practices employed there, interested readers may refer to Spanos (1991) and the book written by May and

The economic design of control charts, as developed by Duncan (1956), forms the basis for building a quality
control plan. Some drawbacks to this design have been pointed out, especially the lack of robust results (Woodall
1986). However, because of the need to balance controls and costs, developments have followed. For example, an
early article discusses quality control skipping (Hsu 1977), which is a plan where, compared to screening, the sample
size is reduced and the control frequency is decreased. The aim of this plan is to find the right control frequency for a
particular cost model. In the field of the economic design of control charts, the model by Lorenzen and Vance (1986)
seems to be a milestone paper, as does the work of Vommi and Seetala (2007), in the sense that a unified model is
presented of the costs incurred during a cycle composed of in-control and out-of-control states. More recently, the
economic design of control charts has been proposed for short-run production (Ho and Trindade 2009).

These articles on control chart design do not refer to the concept of an acceptable limit, nor do the models take
into account the capacity constraints of the control resources.

2.3 Inspection allocation

In the process control literature, the allocation of inspection effort takes into account a fixed control capacity.
Controls can be allocated to minimise one or more of the impact functions related to the failures monitored and the
permeability of the control layers to these failures. Inspection allocation is a field that has been investigated since
Lindsay and Bishop published their study on a cost function per unit produced that takes into account the
inspection cost and the placement of the inspection in the process (Lindsay and Bishop 1964). Since then, several
studies have been conducted. In one of these, which is related to our research interests, Villalobos et al. (1993)
presented a flexible inspection arrangement for serial and multi-staged production systems for printed circuit
boards. They provided a dynamic programming algorithm to optimise global goals (like costs), while taking into
account some local constraints, like inspection tool availability. Verduzco et al. (2001) presented an interesting case
of information inspection allocation, in which they modelled a cost function that takes into account the Type I and
Type II errors linked to each measurement. They simulated their algorithm with a ‘knapsack’ formulation, which
yielded an information-based solution that achieved better performance in terms of classification errors than static
inspection. Their paper has been a source of inspiration for other authors, as they introduced the fact that a control
strategy can be modified based on its gain. More references in the field of inspection allocation can be found in the surveys of Raz (1986), Tang and Tang (1994) and Mandroli et al. (2006).

2.4 Other domains and forms of integration

As indicated above, a number of domains can influence quality control planning, including risk management, statistics, economics and control capacity. In the introduction, we stated that quality control plan design is closely linked to production planning. Although this concept will not be developed further in this section, there is nonetheless an opportunity to enhance models of control capacity allocation and risk limitation.

The authors have also found studies investigating the links between risk and inspection allocation in domains that are totally different from that of semiconductors. In the food industry, for example, inspection allocation is viewed in terms of traceability. While this topic has been widely investigated, we highlight the study of Wang et al. (2010), who designed an inspection plan to both minimise risks and improve operations. In the structural steel industry, Straub (2004) created an inspection plan based on risk modelling. Specifically, he modelled the way cracks and structural failures occur in bridges, and defined a control policy for a particular level of risk acceptance. This work is especially appealing to researchers, as it is the only work on control allocation based on the risk of failure and a level of risk acceptance that is not about monetary cost.

2.5. Literature conclusion

The literature review covers several domains: risk, SPC, inspection allocation and the integration of quality and quantity. We accept that these disciplines are only loosely related, and that few developments encompass all of them. In fact, of the articles included in the review, none describes a global risk insurance model for process control plan design. Nevertheless, the authors of these articles did highlight the following three approaches:

(1) Material at risk (MAR).
(2) The added value of controls.
(3) The allocation process.

The exposure-based process control allocation model developed in the remainder of this article is based on a combination of these three approaches.

3. Problem formulation and the proposed approach

The proposed model is aimed at controlling MAR in the production system. First and foremost, it allocates control capacity over products, processes and tools with a view to reducing uncertainty in the line, and then applies controls where they are needed. At the same time, it computes a feasible insurance limit that can be respected with the current control capacity and manufacturing system performances.

3.1 Notation and assumptions

This model is based on the following notation and assumptions:

**Notation**

- **n** Number of controls.
- **H** Production horizon, which is the quantity of products manufactured during a rolling period.
- **i** Index of the control number.
- **t_i** Date of the i-th control, \( i \in \{1, 2, \ldots, n\} \).
- **X** Vector of the dates of the n controls, \( X=(t_1, t_2, \ldots, t_i, \ldots, t_n) \).
- **R_L** Threshold of risk exposure.
- **MAR^0(t)** Risk function in a reference situation, \( t \in [1, H] \), a zero control situation being considered a reference situation in this model.
- **MAR^0_{max}** Maximum value of MAR reached during the horizon H considered in a zero-control situation, \( MAR^0_{max} = \text{Max}(MAR^0(t)) = H \).
MAR^n_X(t) Risk function at $t$, where $t \in [1, H]$ and $n$ controls are planned during production horizon $H$ at dates defined by $X$; denoted $MAR$ for the sake of simplicity, when no confusion is possible, for a specific tool $z$.

$H_z$ Production horizon for tool $z$.

$MAR^{n_X}_{\text{max}}$ Maximum value of $MAR$ reached during the horizon $H$ considered in a situation where $n$ controls are planned, depending on the positions of these $n$ controls $T = (t_1, t_2, \ldots, t_n)$ along that horizon, $MAR^{n_X}_{\text{max}} = \max\{MAR^n_X(t)\}$.

$AV^{n_X}$ Added value of the $n$ controls expressed in risk savings over the production horizon $H$, $AV^{n_X} = MAR^0_{\text{max}} - MAR^{n_X}_{\text{max}}$.

$\text{TOTALCAPA}$ Available allowed capacity of control in time units.

$C_{ti}$ Control duration of operation $i$.

$CH^i$ Charge of a risk-based control plan, expressed in time units, it is the product of the number of controls and the duration of a single control.

$CPM_i$ Measurement tool capability index when measuring operation $i$.

$CP_i$ Process tool capability index when executing operation $i$.

$Q_j$ Quantity of product $j$ to produce, $j \in \{1, \ldots, NP\}$.

$J(i)$ Set of products where operation $i$ is included in the process flow; example from Table 2: $i = 1$ ($OP1$); $J(i) = \{1, 3\}$.

$NP$ Maximum number of products.

$n^1_i$ Number of controls assigned to operation $i$ at Stage 1 of the $HA$.

$n^2_i$ Number of controls assigned to operation $i$ at Stage 2 of the $HA$.

$n_i$ Number of controls assigned to operation $i$ at the end of the $HA$: $n_i = n^1_i + n^2_i$.

$\text{TOOLS}$ Set of tools used to process different operations in the production plan.

$\text{Tools}(i)$ Subset of qualified process tools on which operation $i$ can be processed.

Assumptions

Assumption 1: The model allocates controls and is not restricted to measurement. A control can be started by measurement or inspection, and can include any actions necessary to relieve uncertainty about products or processes (re-measurement, analysis, out-of-control action plan, local maintenance, sorting or scrapping of products, etc.).

Assumption 2: A control uncovers 100% of the uncertainty since the last control on both products and processes. $\forall t_i \in T, MAR^{n_X}(t_i) - 0$ and $MAR^{n_X}(t_i)^+ = 0$ – see Figure 2.

Assumption 3: There is no delay on a control, relative to the time scheduled in the production plan. The effect of the control is supposed to be immediate, i.e. control duration is negligible relative to the production horizon.

Assumption 4: A control does not show the alpha or beta risks of the associated measurement (the risk of not detecting a bad product, and the risk of identifying a good product as bad respectively).

Assumption 5: The budget allocated for insuring the consequences of control actions is refunded immediately, and all the capacity consumed is automatically offset. The controls have no effect on the risk exposure limit. This is denoted as $RL = \text{Constant}$.

Assumption 6: Every control has a fixed duration. This hypothesis is central to the partition algorithm.

Figure 2. Risk evolution.
Assumption 7: \( MAR^0(0) = 0. \) For this model, the initial risk value is 0.

Assumption 8: The number of controls is an integer. For example, 1.2 controls is considered as either 1 or 2 controls.

This proposition helps process control design to become more rational. It can be understood as an insurance viewpoint for designing layers of control.

The assumptions represent the following case: Production continues until a systematic control is performed on a product. Exceeding the risk exposure limit \( R_L \) could lead to a major disruption (in production, in customer penalties, etc.). A loss below this limit is seen as acceptable, and can be offset. The goal is then to define: (1) a strategy to ensure a minimum number of controls to remain below \( R_L \); and (2) the position of the controls in the production plan. Figure 2 shows how risk increasingly follows production throughput. Whenever a control is performed, it is reset to zero. In this figure, three controls have been planned in order not to exceed \( R_L \).

3.2 Model formulation

The problem of control plan design can be presented as follows:

Objective function

\[
\text{Maximize } \sum_{i=1}^{NOP} n_i \times C_i
\]

Constraints

\[
n_i \in \mathbb{R} \quad \forall i \in \{1, \ldots, NOP\}
\]

\[
n_i \leq \sum_{j \in (j \cup i)} Q_j \quad \forall i \in \{1, \ldots, NOP\}
\]

\[
\sum_{i=1}^{NOP} n_i \times C_t \leq \text{TOTALCAPA}
\]

\[
\sum_{i=1}^{NOP} \frac{n_i}{\text{Card(Tools(i))}} \geq \frac{1}{R_L} \sum_{i=1}^{NOP} \frac{\sum_{j \in (j \cup i)} Q_j}{\text{Card(Tools(i))}} \quad \forall z \in \text{TOOLS}
\]

The objective of this problem formulation is to maximise the overall effectiveness of the planned controls. The criterion \( C_i \) represents the weight of each process operation in terms of this objective. Within the current framework, we try the criterion \( C_i = CPM_i / CP_i \), which seems to reflect the effectiveness of a control with respect to its measurement and process capabilities. Constraint (2) ensures that a sampling rate of 1 for each operation is not exceeded. The capacity constraint is respected using constraint (3), which means that the total control duration (\( \Sigma n_i C_t \)) should be less than or equal to the total available capacity of the control \( \text{TOTALCAPA} \). The objective of remaining below the threshold risk exposure \( R_L \) is taken into account by constraint (4). This threshold is set for the process tools as follows:

\[
\hat{M}AR_{z}^{\text{max}} \leq R_L \quad \forall z \in \text{TOOLS}
\]

with

\[
\hat{M}AR_{z}^{\text{max}} = \frac{\hat{H}_z}{n_z + 1}; \quad \hat{H}_z = \sum_{i=1 \text{Tool}(i) \geq z}^{\text{NOP}} \frac{\sum_{j \in (j \cup i)} Q_j}{\text{Card(Tools(i))}}; \quad \hat{n}_z = \sum_{i=1 \text{Tool}(i) \geq z}^{\text{NOP}} \frac{n_i}{\text{Card(Tools(i))}}
\]

Constraint (4) is central to the development presented in this paper and is the expression of the insurance viewpoint mentioned above.
The threshold $R_L$ is set on the process tools, because we assume that they constitute the most likely sources of failure that can lead to massive product losses. We also assume that quality control on products can be used for process tools qualification.

The focus of this formulation is the decision as to how many controls should be assigned to each operation. It can be applied to non-serial manufacturing systems, and complements traditional approaches that determine the best placement of controls (e.g. Verduzco et al. 2001).

The control plan design problem (CPDP) that is aimed at maximising control effectiveness can be formalised as a 0-1 knapsack problem (0-1KP). The analogy is presented in Table 1. The objective function of the CPDP is to maximise the total control effectiveness, while respecting the fact that the maximum total control time of the chosen number of controls must not exceed the total control capacity $TOTALCAPA$. Every decision variable $n_i$ of CPDP is bounded by the sum of the quantity of products to produce to which the operation $i$ belongs: $\sum_{j \in \mathcal{J}(i)} Q_j$. In fact, the CPDP is closer to what is called bounded knapsack problem (BKP), which can be easily transformed into a 0-1KP. This latter is known to be an NP-hard problem (Garey and Johnson 1979). Since the latter are not solvable optimally in a polynomial time for hard and large instances, heuristic and meta-heuristic methods are usually used as a compromise between computation cost and solution quality. This is why we focus in this paper on finding an HA for the CPDP that can be applied to solving large instances in a reasonable computation time.

Table 1. Knapsack formulation of the control plan design problem.

<table>
<thead>
<tr>
<th>0-1KP</th>
<th>Control Plan Design Problem (CPDP)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td><strong>Data</strong></td>
</tr>
<tr>
<td>$N$: number of items</td>
<td>$NOP$: number of control operations</td>
</tr>
<tr>
<td>$W$: Maximum total weight</td>
<td>$TOTALCAPA$: Total available control capacity</td>
</tr>
<tr>
<td>$p_i \in \mathbb{R}^+$ $\forall i \in {1, \ldots, N}$: profit associated with item $i$</td>
<td>$C_i \in \mathbb{R}^+$ $\forall i \in {1, \ldots, N}$: effectiveness of control operation $i$</td>
</tr>
<tr>
<td>$w_i \in \mathbb{R}^+$ $\forall i \in {1, \ldots, N}$: weight of item $i$</td>
<td>$C_l \in \mathbb{R}^+$ $\forall i \in {1, \ldots, N}$: duration of control operation $i$</td>
</tr>
<tr>
<td><strong>Objective</strong></td>
<td><strong>Objective</strong></td>
</tr>
<tr>
<td>Find a subset $X + {1, \ldots, N}$ such that</td>
<td>Find $n_i$ for each control operation such that</td>
</tr>
<tr>
<td>$\sum_{x \in X} w_x \leq W$</td>
<td>$\sum_{i \in {1, \ldots, NOP}} n_i \times C_i \leq TOTALCAPA$</td>
</tr>
<tr>
<td>and which maximize</td>
<td>and which maximize</td>
</tr>
<tr>
<td>$\sum_{x \in X} p_x$</td>
<td>$\sum_{i \in {1, \ldots, NOP}} n_i \times C_i$</td>
</tr>
<tr>
<td><strong>Decision variables</strong></td>
<td><strong>Decision variables</strong></td>
</tr>
<tr>
<td>$x_i = 1$ if item $i$ is selected; 0 otherwise $\forall i \in {1, \ldots, N}$</td>
<td>$n_i$: number of controls to perform on operation $i$ during the production horizon.</td>
</tr>
</tbody>
</table>

$0 \leq n_i \leq \sum_{j \in \mathcal{J}(i)} Q_j$
3.3 A two-stage heuristic algorithm

To solve the problem, an heuristic in two stages is presented. It is illustrated in Figure 4.

3.3.1 Stage 1: Risk-based minimum control plan

The computation is presented for 1 tool, and the results are extended to the operations and product levels. As the \( MAR \) increases linearly,

\[
MAR_z^T(t_i) = (t_i - t_{i-1}), \quad \forall i \in \{1, \ldots, n + 1\}, \quad z \in \text{TOOLS}
\]

where \( t_0 = 0 \) and \( t_{n+1} = H \), then,

\[
AV^{n,T} = R_{\text{max}}^0 - R_{\text{max}}^n = R_{\text{max}}^0 - \max_j MAR_z^T(t) = R_{\text{max}}^0 - \max\{t_1; (t_2 - t_1); \ldots; (t_n - t_{n-1}); (H_z - t_n)\}
\]

For a given \( RL \), defining a control plan which ensures that \( RL \) is not exceeded can be achieved by defining the number and positions of controls to maximise their added value. Let us note that \( n^* \) is the optimal number of controls and \( T^* \) is the vector of their optimal positions.

With these notations and assumptions, the optimal positions of \( n \) planned controls can be determined when maximising the added value, as follows:

\[
AV^* = \max_T AV^{n,T} = R_{\text{max}}^0 - \min_T \{\max\{t_1; (t_2 - t_1); \ldots; (t_n - t_{n-1}); (H_z - t_n)\}\}.
\]

It can be easily demonstrated that:

- The optimal position of controls is a uniform distribution:
  
  \[
  t_i^* = (t_2^* - t_1^*) = \cdots = (t_n^* - t_{n-1}^*) = (H_z - t_n^*) = \frac{H_z}{n + 1}.
  \]

- The optimal number of controls is given by the formula:
  
  \[
  n^* = \left\lceil \frac{H_z}{RL} - 1 \right\rceil,
  \]

where \( \lceil x \rceil \) is the first integer greater than or equal to \( x \).

At the operations level, the number of controls for each operation is given by the following formula:

\[
n_{i}^\text{min} = \left\lceil \frac{\sum_{j \in \{j: Q_j \neq 0\}} Q_j}{RL} - 1 \right\rceil.
\]

If the total number of planned controls is less than this number (for capacity reasons, for instance), the uncertainty exceeds the exposure limit.

3.3.2 Stage 2: Minimum control plan adjustment based on process and control capabilities

At this stage, allocation based on control capacity is made. The minimum control plan from Stage 1 is incorporated into the control capacity allocation. The control capacity to be distributed is expressed in time units. Depending on \( \text{TOTALCAPA} \) and \( CH^1 \), two cases are possible:

Case 1: \( CH^1 < \text{TOTALCAPA} \) (there is control capacity remaining)

The remaining capacity \( \text{RCAPA} = \text{TOTALCAPA} - CH^1 \) is distributed among operations according to a distribution criterion \( C_i, \forall i \in \{1, \ldots, NOP\} \). In this case, the number of controls for operation \( i \) in the final control plan is \( n_i = n_i^1 + n_i^2 \), where \( n_i^2 \) is the number of complementary controls for operation \( i \) computed in Stage 2. The criterion here is related to process and measurement capabilities and is expressed, in our example, as follows:

\[
C_i = \frac{CPM_i}{CP_i}.
\]
Then, \( RCAPA \) is distributed over all the manufacturing operations using criterion \( C_i \) with an iterative greedy heuristic, as presented in Figure 1, where:

- \( S^0 \) is the initial set of operations \( \{1, \ldots, NOP\} \);
- \( RCAPA^0 \) is the initial remaining capacity \( RCAPA^0 = TOTALCAPA-CH^1 \);
- \( S^k \) is the \( k \)th set of remaining operations at the \( k \)th iteration; and
- \( RCAPA^k \) is the remaining capacity at the \( k \)th iteration.

1. The first step calculates the time allocated to every operation using criterion \( C_i \) weighted by the quantity:

   \[
   \frac{\sum_{j \in R(j)} Q_j}{\sum_{i \in S^{k-1}} (C_i \times \sum_{j \in R(i)} Q_j)}
   \]

   The result, expressed in time units, is given by the formula:

   \[
   PART_i = RCAPA \times \frac{C_i \times \sum_{j \in R(j)} Q_j}{\sum_{i \in S^{k-1}} (C_i \times \sum_{j \in R(i)} Q_j)}.
   \]

2. The second step computes the number of controls to be allocated at the operations level, and is given by the formula:

   \[
   n'_i = \left\lfloor \frac{PART_i}{C_{ti}} \right\rfloor
   \]

   where \([x]\) is the first integer less than or equal to \( x \). This is done in order to respect the capacity constraint. However, as the ratio \( PART_i/C_{ti} \) is not an integer, this allocation generates a left-over capacity. This can also happen in cases where this ratio is less than 1 for every operation, generating no complementary control. To address these issues, iterations counted by \( k \) are performed.

The instruction \( S^k \leftarrow S^k - \text{Arg}(\text{Min}_{i \in S^k} C_i) \) (line 15 of the algorithm in Figure 3) represents the elimination of the operation with the lowest ratio \( Cpm/Cp \), i.e. it is among the best-controlled (highest \( Cp \)) and the most difficult to control (lowest \( Cpm \)).

The instruction in line 6 of the algorithm calculates the distribution of the remaining capacity at the \( k \)th iteration.

Line 7 calculates the number of controls to be added at the \( i \)th operation owing to the allocation of the remaining capacity.

For each operation, the number of controls is enhanced with this new number of controls of the \( k \)th iteration. This process ends when no more operations are available (i.e. \( S^k = \emptyset \)) or \( RCAPA^k = 0 \).

At its completion, this algorithm delivers, for every operation, the total number of controls to be performed under the constraints of risk, capacity and production planning. This is a global control plan to be applied by a second stage of control scheduling.

The choice of a criterion related to the process and measurement tool capabilities (\( Cp \) and \( Cpm \) respectively) could be justified by the following: between two control resources for one process operation, it is more efficient to use the best one, i.e. the control resource with the highest measurement capability. At the same time, it is more efficient to allow more time for the control of the operation to correspond to the reduced process capability, if the operation could be controlled with the same control resources. This criterion can be refined, e.g. by taking into account process and control duration, as in the formulation

\[
C_i^2 = C_i \frac{P_{ti}}{C_{ti}}.
\]

We have not simulated this case, however.

**Case 2:** \( CH^1 > TOTALCAPA \) (there is not enough control capacity)

The over-charge \( OVERCH = CH^1 - TOTALCAPA \) has then to be removed from the allocation resulting from Stage 1. It is taken from the controls on different operations, according to a criterion \( C_i = 1/C_r \). The number of controls on operation \( i \) in the final control plan is \( n_i = n_i^1 - n_i^2 \).
Begin
1 \( S^0 = \{1, \ldots, NOP\} \)
2 \( RCAPA^0 = TOTALCAPA - CH^0 \)
3 \( k = 0 \)
4 While \( RCAPA^k > 0 \) and \( \text{card}(S^k) \neq 0 \) 
5 \( k \leftarrow k + 1 \)
6 \[
\begin{align*}
\text{ PART}^i_k & \leftarrow RCAPA^{k-1} + \sum_{j \in S^k} \left( C_i \times \sum_{j \in S^k} Q_{ij} \right) & \forall i \in S^{k-1} \\
\text{ PART}^i_k & \leftarrow 0 & \forall i \in [S^0 - S^{k-1}] \\
\end{align*}
\]
7 \( n_i^{k, a} \leftarrow \frac{\text{ PART}^i_k}{C_i} & \forall i \in S^0 \\
8 \text{ Forall } i \in \{1, NOP\} \\
9 \text{ If } \sum_{j=1}^{i} n_j^{k, a} \geq \sum_{j \in S^k} Q_{ij} \\
10 \quad n_i^{k, a} \leftarrow \sum_{j \in S^k} Q_{ij} - \sum_{j=1}^{i} n_j^{k, a} \\
11 \quad RCAPA^k \leftarrow RCAPA^{k-1} - \sum_{j \in S^k} C_i \times n_j^{k, a} \\
12 \quad S^k \leftarrow S^k - \{i\} \\
13 \text{ Else} \\
14 \quad RCAPA^k \leftarrow RCAPA^{k-1} - \sum_{j \in S^k} C_i \times n_j^{k, a} \\
15 \quad S^k \leftarrow S^k - \text{Arg Min}_{i \in S^k} C_i \\
16 \text{ End If} \\
17 \text{ EndFor} \\
18 \text{ End While} \\
19 \quad n_i^k \leftarrow \sum_{j=1}^{i} n_j^{k, a} & \forall i \in S^0 \\
20 \quad n_i^k = n_i^{k-1} + n_i^k & \forall i \in S^0 \\
End

Figure 3. Iterative greedy heuristic for Stage 2 of the HA.

Figure 4. The two-stage allocation process.
The algorithm is the same as the one described in Case 1, with the following changes:

- \( RCAPA^0 = \text{OVERCH.} \)
- \( S^k \leftarrow S^k \leftarrow \arg(\min_{i \in S^k} C_i) \) to be focused on an operation that is both well mastered (high \( Cp \)) and difficult to control (low \( Cpm \)).
- In order to remove enough control and gain more capacity, line 9 is modified as follows:
  \[
  \forall i \in S^k, n_{i}^{2,k} = \left\lfloor \frac{\text{Part}_i^k}{Ct_i} \right\rfloor.
  \]

### 4. An illustrative example

#### 4.1 Tested instance

The instance we tested in this illustrative example is presented in Table 2. It represents a production plan composed of three products \( P1, P2 \) and \( P3 \) in quantities 10, 9 and 17 respectively. Four controllable process operations (\( Op1 \) to \( Op4 \)) are used in the manufacturing process using three process tools \( T1, T2 \) and \( T3 \). Each product is characterised by its own process flow, including a subset of the process operations. For example, \( P2 \) has only operations \( Op2 \) and \( Op4 \) in its process flow. Operations are executed on process tools according to the qualification matrix, which indicates, for each operation, the subset of process tools that meet the requirements to execute it. For example, \( Op1 \) can only be executed on tool \( T2 \). Each operation has three characteristics: the control time \( Ct \), i.e. time needed to control the quality of the operation after execution, the process capability index \( Cp \) and the measurement capability index \( Cpm \).

#### 4.2 Resolution with heuristic approach

The resolution using the HA can be easily performed using tabular software. The various resolution steps are explained in detail in this subsection.

**Stage 1: Risk-based control plan**

This stage starts by computing a forecast value of the process tool horizons using the following formula:

\[
\hat{H}_i = \sum_{i=1}^{4} \sum_{j \in \mathcal{R}_i} \frac{Q_j}{\text{Card}(\text{Tools}(i))}
\]

\[
\hat{H}_{T1} = \frac{36}{2} + \frac{10}{3} = \frac{64}{3} = 21.33
\]

\[
\hat{H}_{T2} = \frac{27}{1} + \frac{10}{3} + \frac{26}{2} = \frac{130}{3} = 43.33
\]

\[
\hat{H}_{T3} = \frac{36}{2} + \frac{10}{3} + \frac{26}{2} = \frac{103}{3} = 34.33
\]

<table>
<thead>
<tr>
<th>( Q_i )</th>
<th>Product</th>
<th>( Op1 )</th>
<th>( Op2 )</th>
<th>( Op3 )</th>
<th>( Op4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( P1 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>( P2 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>( P3 )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Qualification matrix</th>
<th>( T1 )</th>
<th>( T2 )</th>
<th>( T3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{i} ) = 7</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( Ct )</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>( Cp )</td>
<td>1</td>
<td>3</td>
<td>0.5</td>
</tr>
<tr>
<td>( Cpm )</td>
<td>51</td>
<td>40</td>
<td>.5</td>
</tr>
</tbody>
</table>

**Table 2.** Data for the illustrative example.
Then, the minimum number of controls required for each process tool is computed as follows:

\[ n_i^1 = \left\lfloor \frac{H_t}{R_L} - 1 \right\rfloor \quad \forall t \in \{T1, T2, T3\} \]

\[ n_{T1}^1 = \left\lfloor \frac{H_{T1}}{R_L} - 1 \right\rfloor = \left\lfloor \frac{21.33}{7} - 1 \right\rfloor = \lfloor 2.04 \rfloor = 3 \]

\[ n_{T2}^1 = \left\lfloor \frac{H_{T2}}{R_L} - 1 \right\rfloor = \left\lfloor \frac{43.33}{7} - 1 \right\rfloor = \lfloor 5.19 \rfloor = 6 \]

\[ n_{T3}^1 = \left\lfloor \frac{H_{T3}}{R_L} - 1 \right\rfloor = \left\lfloor \frac{34.33}{7} - 1 \right\rfloor = \lfloor 3.90 \rfloor = 4 \]

Then, \( n_{Op1}^1, n_{Op2}^1, n_{Op3}^1 \) and \( n_{Op4}^1 \) are computed as follows:

\[ n_{Op1}^1 = \left\lfloor \left( \sum_{j \in J} Q_j \right) \times \max_{i \in \text{Tools}(\Omega)} \left( \frac{1}{R_L} - \frac{1}{H_t} \right) \right\rfloor \]

\[ n_{Op1}^1 = 27 \times \left( \frac{1}{7} - \frac{1}{H_{T2}} \right) = 27 \times \frac{3}{21.33} = 4 \]

\[ n_{Op2}^1 = 36 \times \max \left\{ \left( \frac{1}{7} - \frac{1}{H_{T1}} \right), \left( \frac{1}{7} - \frac{1}{H_{T2}} \right), \left( \frac{1}{7} - \frac{1}{H_{T3}} \right) \right\} = 6 \]

\[ n_{Op3}^1 = 10 \times \max \left\{ \left( \frac{1}{7} - \frac{1}{H_{T1}} \right), \left( \frac{1}{7} - \frac{1}{H_{T2}} \right), \left( \frac{1}{7} - \frac{1}{H_{T3}} \right) \right\} = 2 \]

\[ n_{Op4}^1 = 26 \times \max \left\{ \left( \frac{1}{7} - \frac{1}{H_{T2}} \right), \left( \frac{1}{7} - \frac{1}{H_{T3}} \right) \right\} = 4 \]

The control charge from Stage 1 is computed:

\[ CH^1 = \sum_{i=1}^{N_{Op}} n_{Op_i}^1 \times C_i = 4 \times 5 + 6 \times 3 + 2 \times 2 + 4 \times 4 = 58. \]

The remaining capacity is \( RCAPA = TOTALCAPA - CH^1 = 70 - 58 = 12 \), which has to be partitioned in Stage 2, according to the criterion

\[ C_i = \frac{C_{pm_i}}{C_{pi}}. \]

**Stage 2:** Allocation of the remaining capacity

This is an iterative greedy heuristic, which allocates the remaining capacity to different operations, according to the criterion \( C_i \). At every iteration, the operation having the lowest criterion is removed from the set of operations concerned with local allocation. Iterations are repeated until all the capacity has been allocated (\( RCAPA^k = 0 \)), or until the set of candidate operations is empty (\( Card(S^k) = 0 \)).

- **Step 1 (Iteration 1)**

\[ RCAPA^1 = RCAPA^0 = 12 \]

\[ S^1 = S^0 = \{Op1, Op2, Op3, Op4\} \]

\[ Part_{Op1} = RCAPA^1 \times \frac{C_{Op1} \times \sum_{j \in J} Q_j}{\sum_{i=1}^{4} C_i \times \sum_{j \in J} Q_j} = 12 \times \frac{5 \times 27 + 0.33 \times 36 + 0.33 \times 36 + 8 \times 10 + 0.07 \times 26}{5 \times 27 + 0.33 \times 36 + 0.33 \times 36 + 8 \times 10 + 0.07 \times 26} = 7.08 \]

\[ Part_{Op2} = 12 \times \frac{0.33 \times 36}{228.85} = 0.63 \]

\[ Part_{Op3} = 12 \times \frac{8 \times 10}{228.85} = 4.19 \]

\[ Part_{Op4} = 12 \times \frac{0.07 \times 26}{228.85} = 0.1 \]
\[ n_{Op1}^{2-\text{iter1}} = \left[ \frac{7.07}{5} \right] = 1 \]
\[ n_{Op2}^{2-\text{iter1}} = \left[ \frac{0.63}{3} \right] = 0 \]
\[ n_{Op3}^{2-\text{iter1}} = \left[ \frac{4.19}{2} \right] = 2 \]
\[ n_{Op4}^{2-\text{iter1}} = \left[ \frac{0.1}{4} \right] = 0 \]

- **Step 2 (Iteration 2)**

\[ \text{Arg Min}_{i \in S_2^1} C_i = 4 \]

\( Op4 \) is removed from the set \( S_2^1 = \{ Op1, Op2, Op3, Op4 \} \).

\( S_2^2 = \{ Op1, Op2, Op3 \} \)

The remaining control capacity from Step 1 is:

\[ RCAPA_2^2 = RCAPA_1^2 - \sum_{i \in \{1, \ldots, NOp\}} C_i \times n_{Opi}^{2-\text{iter1}} = 12 - 9 = 3 \]

\[ \text{Part}_{Op1} = 3 \times \frac{5 \times 27}{5 \times 27 + 0.33 \times 36 + 8 \times 10} \approx 1.78; \quad n_{Op1}^{2-\text{iter2}} = \left[ \frac{1.78}{5} \right] = 0 \]

\[ \text{Part}_{Op2} = 0.16; \quad n_{Op2}^{2-\text{iter2}} = 0 \]

\[ \text{Part}_{Op3} = 1.05; \quad n_{Op3}^{2-\text{iter2}} = 0 \]

The remaining capacity is 3.

- **Step 3**

\[ \text{Arg Min}_{i \in S_2^2} C_i = 2 \]

\( S_3^3 = \{ Op1, Op3 \} \)

\[ RCAPA_3^3 = RCAPA_2^2 = 3 \]

\[ \text{Part}_{Op1} = 3 \times \frac{5 \times 27}{5 \times 27 + 8 \times 10} = \frac{405}{215} = 1.88; \quad n_{Op1}^{2-\text{iter3}} = \left[ \frac{1.88}{5} \right] = 0 \]

\[ \text{Part}_{Op3} = 3 \times \frac{8 \times 10}{5 \times 27 + 8 \times 10} = 1.11; \quad n_{Op3}^{2-\text{iter3}} = \left[ \frac{1.11}{2} \right] = 0 \]

- **Step 4**

\[ \text{Arg Min}_{i \in S_3^3} C_i = 1 \]

\( S_4^4 = \{ Op3 \} \)

\[ RCAPA_4^4 = RCAPA_3^3 = 3 \]

\[ \text{Part}_{Op3} = 3; \quad n_{Op1}^{2-\text{iter4}} = \left[ \frac{3}{2} \right] = 1 \]
Stage 2 is stopped, and the remaining capacity remaining at the end of Step 4 is 1.

**Results synthesis**

**Stage 1:**

<table>
<thead>
<tr>
<th>Op1</th>
<th>Op2</th>
<th>Op3</th>
<th>Op4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1^i$</td>
<td>4</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

**Stage 2:**

<table>
<thead>
<tr>
<th>$n_i^{\text{t-rank}}$</th>
<th>Op1</th>
<th>Op2</th>
<th>Op3</th>
<th>Op4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iter1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Iter2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Iter3</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Iter4</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$n_i^2$</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

**Final results:**

<table>
<thead>
<tr>
<th>Op1</th>
<th>Op2</th>
<th>Op3</th>
<th>Op4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_i$</td>
<td>$n_1^i$ + $n_2^i$</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

**Objective function:**

$$
\sum_{i \in \{1, \ldots, NOP\}} C_i \times n_i = 5 \times 5 + 6 \times 0.33 + 5 \times 8 + 4 \times 0.07 = 67.29
$$

**MAR values**

As this algorithm provides a predicted quality control plan, the expected best MAR values on the various process tools are computed as follows:

$$
\hat{\text{MAR}}_{T_1}^{\text{max}} = \frac{\sum_{i=1}^{4} \frac{Q_i}{\text{Card}(\text{Tools}(i))}}{1 + \frac{\sum_{i=1}^{4} \frac{Q_i}{\text{Card}(\text{Tools}(i))}}{\text{Card}(\text{Tools}(i))}} = \frac{\frac{36}{1} + \frac{10}{2} + \frac{6}{3}}{1 + \frac{\frac{36}{1} + \frac{10}{2} + \frac{6}{3}}{3}} = \frac{64}{17} = 3.76 \approx 4
$$

$$
\hat{\text{MAR}}_{T_2}^{\text{max}} = \frac{\frac{27}{1} + \frac{10}{2} + \frac{26}{3} + \frac{2}{4}}{1 + \frac{\frac{27}{1} + \frac{10}{2} + \frac{26}{3} + \frac{2}{4}}{4}} = \frac{130}{29} = 4.48 \approx 5
$$

$$
\hat{\text{MAR}}_{T_3}^{\text{max}} = \frac{\frac{36}{1} + \frac{10}{2} + \frac{26}{3} + \frac{2}{4}}{1 + \frac{\frac{36}{1} + \frac{10}{2} + \frac{26}{3} + \frac{2}{4}}{4}} = \frac{103}{23} = 4.48 \approx 5
$$

**4.3 Comparison with the optimal solution**

The optimal solution is obtained using the IBM ILOG CPLEX Optimisation Studio with the formulation presented in Section 3.2.

It is obtained in a negligible CPU time, as the number of operations is small. The results obtained are given below:

- Optimal control numbers

<table>
<thead>
<tr>
<th>Op1</th>
<th>Op2</th>
<th>Op3</th>
<th>Op4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_i$</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>
Optimal objective function: 86.264

This control plan leads to the following predicted \( \text{MAR} \) values of process tools:

\[
M_{\text{AR}}^{\text{max}} T_1 = \frac{\sum_{i=1}^{4} n_{\text{OP}i} \cdot \frac{Q_i}{\text{Card}(\text{Tool}(i))}}{1 + \sum_{i=1}^{4} n_{\text{OP}i} \cdot \frac{C_2}{\text{Card}(\text{Tool}(i))}} = \frac{\frac{36}{2} + \frac{10}{3}}{1 + \frac{5}{2} + \frac{8}{3}} = \frac{64}{20} = 3.2 \approx 4
\]

\[
M_{\text{AR}}^{\text{max}} T_2 = \frac{27}{1 + \frac{4}{2} + \frac{4}{3} + \frac{2}{3}} = \frac{130}{29} = 4.48 \approx 5
\]

\[
M_{\text{AR}}^{\text{max}} T_3 = \frac{36}{2} + \frac{10}{3} + \frac{26}{1 + \frac{6}{2} + \frac{8}{3}} = \frac{103}{26} = 3.96 \approx 4
\]

As presented in Table 3, the performance of the \( HA \) relative to that of the optimal solution for this example is about 22%. The remaining control capacity after allocation using the \( HA \) is equal to 1, and to zero for \( OA \). The expected \( \text{MAR}^{\text{max}} \) value of each process tool respects the threshold exposure limit \( R_L \) in the two approaches, with a better value for \( T_3 \) with \( OA \). In fact, the lower the \( \text{MAR}^{\text{max}} \) of a process tool, the lower the potential product loss caused by the tool.

Further experiments or a worst case study are needed to draw conclusions about the relative performance of the proposed heuristic.

5. Conclusion and perspectives

In this paper, we have presented an approach for control plan design which is aimed at managing risk exposure and maximising quality control effectiveness, which is evaluated with a criterion related to measurement and process capability indices.

The proposed algorithm consists of two stages. The first stage allocates controls to keep the exposure to risk below a threshold, expressed as the number of successive products processed without any control. This stage serves as insurance for manufacturing management. The second stage is an iterative greedy heuristic which adjusts the control plan defined in the first stage. In the case where Stage 1 leads to a control charge that is less than the available capacity, the remaining capacity is allocated in Stage 2 to the operations requiring it most (the highest \( \text{Cpm/Cp} \) ratio and the highest volumes in the first stage). In the case of under-capacity, some controls are removed from the operations with the least effective control (the lowest \( \text{Cpm/Cp} \) ratio and the lowest volume). An illustrative example was presented, and the results of the heuristic algorithm are compared to the optimal solution.

Several perspectives can be helpful in continuing this research:

- The first addresses the stability of the allocation algorithm. For a given context, the allocation generates controls that influence the parameter allocation criteria, like \( Cp, Cpm \).

- The second can integrate new allocation criteria, including new parameters like processing time and measurement time

\[
C^2 = \frac{Cpm \cdot Pt}{Cp \cdot Ct}
\]

- The third could be to allow rational numbers to be assigned to controls. In this case, planning would span over several production horizons. For instance, 1.2 controls would be planned as two controls over five production horizons.
Finally, it will be interesting to investigate the robustness of the algorithm in terms of the variation in the initial conditions (especially a change of $Q_j$, or a modification of $C_p$) in a full model integrating the stability issue and other enhanced criteria.

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Note
1. In semiconductor manufacturing, the term product can mean either wafer or chip, depending on whether it refers to front-end or back-end manufacturing. In this document, the products are wafers.

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