Impact of type-II inspection errors on a risk exposure control approach based quality inspection plan

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This paper studies the effect of type-II inspection errors on the effectiveness of a quality inspection plan designed utilizing a risk exposure control approach. To do so, the probability of type-II error is integrated into the Material At Risk (MAR) model used to control risk exposure. A linear programming formulation, including the stochastic behaviour of the model, is proposed and solved. Experiments conducted to analyze the effect of inspection error on risk exposure control reveal the computational complexity of the problem. Moreover, the impact of type-II inspection error has been found to be more significant when it becomes high, and then the quality control plan should be adjusted according to the desired confidence level that corresponds to the probability of being above a given risk-exposure threshold. A 3-step method is proposed to give some insights into how to take into account this error, in order to consolidate the risk exposure control approach and design more effective inspection plans.

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1. Introduction

The research presented in this paper is an extension of the model of quality inspection planning based on risk exposure control approach, proposed in Bettayeb et al. [3]. The model seeks to add an insurance perspective to quality control planning, by embedding a simplified model of the MAR concept used to control the amount of quality uncertain products (risk exposure), while judiciously allocating the available inspection capacity over a finite planning horizon. This paper refines the model of MAR by taking into account type-II inspection error and analyses its impact on the risk exposure control approach based quality inspection plan. These works lie within the general scope of quality assurance, and more specifically the planning of quality control activities.

In quality assurance, the quality control plan focuses on process outputs, and has two main objectives: variability reduction, and the prevention and control of events that initiate massive losses related to quality issues.

The prevention of massive product recalls, returns, and scraps is important to manufacturers, as even one occurrence of such an event can affect an entire production run, impact customers, ruin customer goodwill, and generate financial penalties or lawsuits, or both.

The MAR concept, which was first introduced by Bean [1] for the semiconductor industry, seems very appropriate in this context. It refers to the production of potentially faulty products, which is directly influenced by production and quality control plans. It can also be strongly influenced by operations management, and particularly by the quantity produced between two controls.

The search for a way to reduce the amount of MAR in manufacturing can be seen as an opportunity to incorporate a means to monitor, control, and manage massive scraps into the traditional approaches to production line assessment.

This concept is used by Bettayeb et al. [3,4] to design and evaluate quality control plans from an insurance perspective. A generic model of MAR and its usefulness in a risk exposure control approach based quality inspection plan is proposed in Bettayeb and Bassetto [2]. Their objective is to judiciously allocate inspection capacity, in order to maximize the effectiveness of quality control activities while guaranteeing a minimum level of risk exposure. However, the proposed method assumes error-free inspection in the MAR model. If we can suppose that corrective actions are always beneficial for the system, even if they are initiated following a false alarm (at least the system is as good as it was before), type-II errors would then be more harmful in terms of actual losses. This is because the actual losses will continue to increase if a failure has occurred since

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the last inspection and it is not detected by the current inspection, while the MAR will have been incorrectly reset to zero. In fact, the probability of type-II error makes the MAR stochastic as illustrated in Fig. 1. After each inspection, the MAR continues to increase with a probability \( \beta \) (inspection does not detect failure and no action on the process is taken) and falls to zero with a probability \( 1 - \beta \) (inspection generates a true detection signal, then the process is immediately corrected if necessary, and the actually lost products are recovered (re-planning, rework, outsourcing, etc.)).

The consequences of type-II error on the risk exposure control approach are illustrated by the example in Fig. 2. The MAR proposed model in [3] issues the inspection plan, as shown in Fig. 2(a). In this model, there is no possibility of exceeding the \( R_L \) threshold. The constraint generated by the prohibited zone is always respected. By considering \( \beta \neq 0 \), as illustrated in Fig. 2(b), there is a non null probability of being within the prohibited zone. For instance, if the second control (at \( t_2 \)) fails, the MAR exceeds \( R_L \) and reaches level C. Then, if the third control (at \( t_3 \)) also fails, the MAR reaches level B, far exceeding \( R_L \). When \( \beta = 0 \), the bold dashed path has a non-null probability of being created.

The aim of this paper is to integrate type-II inspection error into the model MAR model and to investigate its effect on the optimized quality control plan from an insurance perspective. The remainder of the paper is organized as follows. Section 2 reviews the works related to this research. The proposed model, with its assumptions and mathematical formalization is presented in Section 3. The experiments are presented and discussed in Section 4. The last section presents our concluding remarks and addresses possible directions for future work.

### 2. Literature review

Designing an adequate quality inspection plan to prevent massive scraps from being produced encompasses several classes of problem. A first interesting quality control planning problem, faced in the early phase of production system design, lies in the selection of locations and machines for inspection activities. Once the production system and inspection machines established, the second quality control planning problem lies in the operational management of quality control activities (inspection, data analysis, corrective actions) and their integration with other production activities.

Since control resources are generally limited, they must be allocated judiciously. Some five decades ago, Lindsay and Bishop [9] developed an economic algorithm for inspection allocation, based on a cost function per unit produced, taking into account the inspection cost and its location in the process. Since this paper was published, the field of inspection allocation has become an area of intense research. For a clear picture of this research field, authors recommend the surveys of Raz [13], Tang and Tang [15] and Mandrol et al. [10].

Some of the remarkable achievements in terms of improving production reliability have been of particular importance. Villalobos et al. [17] introduce and model the concept of flexible inspection system (FIS) for serial multi-stage production systems in the field of printed circuit boards. They provide a dynamic programming algorithm to optimize global goals (like costs), while taking into account some local constraints, like inspection tool availability. In automated visual inspection, Verduzco et al. [16] present an interesting case of information-based inspection allocation. They modelled a cost function taking into account type-I and type-II errors, and the information gain obtained by each measurement. They formulated the inspection allocation problem as a Knapsack Problem (KP), and proposed a greedy algorithm to solve it. Their results reveal that the information-based solution outperforms static inspections, in terms of classification errors. The information gain concept is analogous to the MAR: the more the total information gain, the less the expected amount of defective products is.

The works of Rabinowitz and Emmons [12] and Emmons and Rabinowitz [8] present an inspiring nonlinear model of the inspection allocation problem. The objective function is to maximize the expected fraction of good items produced by optimizing the inspections scheduling. They introduce the time passed since the last inspection and use it to express the proportion of defect-free items. This formalization is strongly related to the MAR concept.
They assume a two-state processing stages (machines): (i) up – all processed items during this stage do not acquire any defect at this stage, or (ii) down – all items processed by the machine acquire a defect. However, the model supposed that inspections give a perfect information (error-free) about the state of stage.

Inspection allocation problems have also been studied in connection with several social and economic sectors. For instance, in maritime traffic, the inspection of cargo containers at the port of entry has been rationalized, with the objective of better controlling risks related to security and fraud issues while minimizing costs and waiting times (e.g., Elsayed et al. [7], McClay and Dreding [11]). In the structural steel industry, an inspection model based on risk modelling has been presented by Straub and Faber [14]. They investigated the way cracks and failures occur in bridges, and defined a control policy based on the acceptance of a particular level of risk. This work has been especially inspirational for us, as it is the only work on control allocation based on the risk of failure and a risk acceptance level, rather than on a general cost function.

Moreover, with the development of sensor technologies and the growth in their use have emerged several strategic and operational decision making issues which are very close to those of inspection allocation. These involve selecting appropriate sensors, determining their optimal locations, and deciding when and how data are collected and utilized (Ding et al. [6,5]). The objective, which is always reduced to a cost function, is to maximize the intake of information in terms of the diagnosis and detection of failures in the field monitored.

To summarize, inspection allocation problems can be divided into two categories: detection/monitoring-oriented, and diagnosis-oriented. They can usually be formalized as an optimization process, where the objective function is either the total or the unit expected costs. Constraints are generally related to the intrinsic characteristics of manufacturing system in question, such as the type of production configuration, the type of inspection, and the type of defect. Several other constraints are also considered, such as the Average Outgoing Quality Limit (AOQL) and the budget limit.

3. The model

To take type-II errors into account in quality control planning from an insurance point of view, we need to examine a finite set of MAR paths, each of which corresponds to a combination of successful and failed planned inspections occurring during the planning horizon. The MAR paths can be enumerated using a decision tree, as illustrated in the example in Fig. 3. Note that the increase in the number of paths (denoted by \(p\)) is exponential with the number of planned inspections denoted by \(n\); \(p \in \{1, \ldots, 2^n\}\).

3.1. Assumptions and notations

Process and quality controls are based either on information obtained by inspecting the process parameters during the various process steps, or from the quality characteristics of semi finished or finished products. The proposed model is designed to take into account the \(\beta\)-risk (type-II error, or consumer risk) of non detecting process drifts or failures that may impact all subsequent products until they are detected and the processing tool is fixed.

The developments presented in this paper are valid for all failure modes, the occurrence of any of which will, at a minimum, bring the process to an out-of-control state.

Let \(R_0\) be the threshold of risk exposure that should never be exceeded if a major disturbance of the production organization is to be avoided. This means that the production organization could manage an actual loss below \(R_0\), as revealed the measurement result. If the actual loss exceeds \(R_0\), the organization will need significant resources to compensate for the delays caused, and perhaps to recall products already delivered and compensate affected customers. Control plans are usually designed to mitigate risks related to process or tool failures, which are commonly the causes of major product losses.

The control plan for a subsystem specifies the way in which each risk is monitored and controlled, depending on its ranking and a predefined threshold of risk acceptance. We propose a risk-based control plan design that will be suitable for either an entire production system or one of its subsystems, or a single production entity like a tool – an assembly device, for instance. The planning horizon, which refers to the number of products to be processed, will be denoted by \(H\) in the remainder of this paper.

Let \(x\) be a quality and/or process inspection plan characterized by the triplets \((n^*, \tau^*, \gamma^*)\) where:

- \(n^*\) the total number of inspections planned during the planning horizon \(H\)
- \(\tau^*\) the vector of inspection dates, \(\tau^* = \{t_1, \ldots, t_L\}\), where \(t_L\) refers to the rank of the processed product after which the \(k\)-th inspection will be performed, with: \(1 \leq t_1 < t_2 < \cdots < t_k < \cdots < t_L \leq H\)
- \(\gamma^*\) the vector of independent and identically distributed (i.i.d.) binary random variables indicating the success (1) or the failure (0) of each inspection in the inspection plan \(x\), it corresponds to a Bernoulli trials process with \(\beta\) the probability of failure.
- \(\gamma^*_p\) is a binary random variable which represents the state of success or failure of the \(k\)-th inspection.
- \(\gamma^*\) the vector indicating the combinations of success and failure of the \(n^*\) inspections of the MAR path \(p\). It is the \(p\)-th possible outcome of the random vector \(\gamma^*\). Each inspection having two possible performances (success or failure), the sample space of \(\gamma^*\) is finite and is composed of \(P^* = 2^{n^*}\) possible outcomes, each of them corresponds to a specific path of the MAR and is indexed by \(p \in \{1, \ldots, P^*\}\).
- \(\gamma^*_p\) is a binary sequence, where \(\gamma^*_p = 1\) if the \(k\)-th inspection in the \(p\)-th MAR path is successful (type-II error-free); 0 otherwise. In this paper, it is obtained by decimal-to-binary conversion of the quantity \((P^* - p)\) using \(n^*\) bits.
- \(\beta\) the probability of inspection failure (type-II inspection error), with: \(\beta = Pr(\gamma^*_p = 0) = 1 - Pr(\gamma^*_p = 1)\).
3.2. Mathematical formalization

Knowing the probability of making a type-II error at each inspection, the objective is to minimize the overall probability of the quality risk exposure (MAR) over a predefined threshold ($R_k$), i.e. the probability that the MAR overlaps the threshold $R_k$ (prohibited zone in Fig. 2(b)).

Minimize $\sum_{p=1}^{P} Pr(\text{MAR}^p \text{ overlaps } R_k)$ \hspace{1cm} (1)

This objective function makes it possible to determine, for a fixed number of inspections, their optimal positions in the planning horizon such that the probability of being exposed to a quality risk exceeding $R_k$ is minimized. The probability of occurrence of the $p$th MAR path in inspection plan $x$ can be computed as follows:

$Pr(\{Y^x = y_p^x\}) = Pr(\{Y_1^x = y_{p,1}^x, \ldots, Y_k^x = y_{p,k}^x, \ldots, Y_n^x = y_{p,n}^x\})$

$= \prod_{k=1}^{n} \beta(1 - y_{p,k}^x) + (1 - \beta)y_{p,k}^x$ \hspace{1cm} (2)

In order to check whether or not the MAR path $p$ exceeds $R_k$, it is sufficient merely to check the value of the MAR at the times of inspection (MAR peaks) and the MAR at the end of the planning horizon $\text{MAR}^p(H)$. The MAR peaks can be expressed recursively for all $k \in \{1, \ldots, n^p + 1\}$, as presented in Eq. (3). At each inspection time $t_k$, the value of the MAR peak depends on the distance $t_k - t_{k-1}$ and the success or failure of the previous inspection at $t_{k-1}$. If the $k$th inspection is type-II error-free (successful, or $y_{p,k}^x = 0$), then the $k$th MAR peak ($\text{MAR}(t_k)$) will be equal to the distance ($t_k - t_{k-1}$). If the $k$th inspection fails ($y_{p,k}^x = 1$), the $k$th MAR peak will be equal to the distance ($t_k - t_{k-1}$) plus the value of the MAR path at $t_{k-1}$:

$\text{MAR}^p(t_k) = (1 - y_{p,k-1}^x)\cdot \text{MAR}^p(t_{k-1}) + (t_k - t_{k-1})$

$= (t_k - t_{k-1}) + \sum_{j=0}^{k-1} (t_j - t_{j-1}) \prod_{m=j}^{k-1} (1 - y_{p,m}^x)$ \hspace{1cm} (3)

where $t_0 = 0$ and $t_{n+1} = H$.

The problem is mathematically formalized by an Integer Linear Program (ILP), as described below. The notation is simplified by omitting the index $x$ corresponding to the inspection plan, when there is no possibility of confusion. The variables, objective function and constraints are expressed as follows:

- **Decision variables**
  Decision variables are defined to determine the position of each inspection $k \in \{1, \ldots, n\}$ in $H$:

  \[ u_{i,k} = \begin{cases} 1 & \text{if } t_k = i; \\ 0 & \text{otherwise.} \end{cases} \hspace{1cm} (4) \]

- **Intermediate variables**
  In order to characterize and evaluate the stochastic behaviour of the MAR when taking into account type-II error made during inspection, some intermediate variables are needed. These variables depend on the decision variables ($u_{i,k}$) and the combinations of success and failure of the inspections, which are represented by the various MAR paths.

- $t_k$: the $k$th inspection time, which corresponds to the index of the item to be inspected and is expressed as follows:

  \[ t_k = \sum_{i=1}^{H-1} i \cdot u_{i,k} \quad \forall k = 1, \ldots, n \hspace{1cm} (5) \]

- $P$: the number of MAR paths, which corresponds to the number of all possible combinations regarding the state of success or failure of each inspection; $P = 2^n$.

- $y_{p,k}$: the state of success or failure of the $k$th inspection of path $p \in \{1, \ldots, P\}$

- $v_{p,k}$: the effect of success or failure of the $k$th inspection of path $p$ on the subsequent MAR peak of that path $\forall p = 1, \ldots, P, \forall k = 1, \ldots, n$.

- **Objective function** The aim is to minimize the overall probability of violating the constraint of the threshold $R_k$, i.e. the sum of probabilities of the paths $p$ of the MAR for which $w_p = 1$.

  Minimize $\sum_{p=1}^{P} w_p \prod_{k=1}^{n} \beta(1 - y_{p,k}) + (1 - \beta)y_{p,k}$ \hspace{1cm} (10)

  The quantity $\prod_{k=1}^{n} \beta(1 - y_{p,k}) + (1 - \beta)y_{p,k}$ corresponds to the probability that the MAR path $p$ occurs. Each MAR path is the realization of a Bernoulli process composed of a sequence of successes and failures of the $n$-inspections.

- **Constraints**
  Constraints, presented below, are used to warrant the integrity of the decision variables (Eqs. (11)--(13)) and to express, algebraically, that certain logical statements are true (Eqs. (16)--(20)). Eq. (15) implies that, at a given time, it is not possible to inspect more products than have been produced up to that time. Eq. (16) warrants that each product can be inspected once at most. Eq. (17) signifies that an inspection is used to inspect only one product. Eqs. (18) and (19) are used to determine $v_{p,k}$, the value of which depends on the validity of the logical statement: $\text{MAR}^p(t_k)$ exceeds $R_k$. In fact, if the last statement is true, then $m$ equals a very small positive real ($m = \epsilon^*$) and then $v_{p,k} = 1$, because Eq. (18) $\Rightarrow v_{p,k} > \epsilon^*$ and Eq. (19) $\Rightarrow v_{p,k} \leq 1 + \epsilon^*$. In the other case ($\text{MAR}^p(t_k) \leq R_k$), $m$ equals a very small negative real ($m = -\epsilon^*$) and then $v_{p,k} = 0$, because Eq. (18) $\Rightarrow v_{p,k} > \epsilon^*$ and Eq. (19) $\Rightarrow v_{p,k} \leq 1 - \epsilon^*$. Finally, Eq. (20) expresses the statement that verifies whether or not MAR path $p$ has at least one peak that exceeds $R_k$.

  \[ u_{i,k} \in \{0, 1\} \quad \forall i = 1, \ldots, H - 1 \quad \forall k = 1, \ldots, n \hspace{1cm} (11) \]
where $m = \frac{MAR_p(i_{p,k}) - R_l}{M}$, with: $M$ a large positive integer.

3.3. Complexity

The problem studied here has a complexity of $O(C^{H-1}_n \cdot 2^n)$, where $C^{H-1}_n = \binom{H-1}{n} = \frac{(H-1)!}{n!(H-n-1)!}$. For a given number of inspections $n < H$, there are $C^{H-1}_n$ possibilities of choosing the inspection dates $t_k(k \in \{1, \ldots, n\})$, and there are $2^n$ MAR paths corresponding to each of them. Each MAR path $MAR_p$ has a probability $Pr(Y = y_p)$ of containing a MAR peak that exceeds $R_l$.

3.4. Illustration with 3 inspections

To illustrate the purpose of our method, let us take an example with 3 inspections to be executed during a planning horizon of 12 runs, applying two different inspection plans $x_1$ and $x_2$ as illustrated in Fig. 4. The threshold $R_l$ is set to 6.

The corresponding decision variables matrices ($U^{x_1}$ and $U^{x_2}$) and intermediary variables ($y$, $V^{x_1}$, $V^{x_2}$, $W^{x_1}$ and $W^{x_2}$) are presented below:

$U^{x_1} = \begin{pmatrix}
    u_{1,1} & u_{1,2} & u_{1,3} \\
    u_{2,1} & u_{2,2} & u_{2,3} \\
    \vdots & \vdots & \vdots \\
    u_{12,1} & u_{12,2} & u_{12,3}
\end{pmatrix}$

$V^{x_1} = \begin{pmatrix}
    v_{1,1,1} & v_{1,1,2} & v_{1,1,3} \\
    v_{1,2,1} & v_{1,2,2} & v_{1,2,3} \\
    \vdots & \vdots & \vdots \\
    v_{12,1,1} & v_{12,1,2} & v_{12,1,3}
\end{pmatrix} = \begin{pmatrix}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & 0 \\
\end{pmatrix}$

$U^{x_2} = \begin{pmatrix}
    u_{1,1} & u_{1,2} & u_{1,3} \\
    u_{2,1} & u_{2,2} & u_{2,3} \\
    \vdots & \vdots & \vdots \\
    u_{12,1} & u_{12,2} & u_{12,3}
\end{pmatrix}$

$V^{x_2} = \begin{pmatrix}
    v_{1,1,1} & v_{1,1,2} & v_{1,1,3} \\
    v_{1,2,1} & v_{1,2,2} & v_{1,2,3} \\
    \vdots & \vdots & \vdots \\
    v_{12,1,1} & v_{12,1,2} & v_{12,1,3}
\end{pmatrix} = \begin{pmatrix}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & 0 \\
\end{pmatrix}$

$W^{x_1} = (0 0 1 1 0 0 1 1) \quad \text{and} \quad W^{x_2} = (0 0 0 1 0 0 1 1)$

In matrices $U^{x_1}$ and $U^{x_2}$, corresponding to inspection plans $x_1$ and $x_2$ respectively, the number of columns equals the number of inspections, and the number of lines corresponds to all their possible positions during the planning horizon ($H - 1$). For example, within inspection plan $x_1$ (respectively, $x_2$), the first inspection is planned at the second run (respectively, the third run), i.e. $u_{1,1} = 1$ (respectively, $u_{1,2} = 1$). The matrix $y$ represents the sample space of the both random binary vectors $Y^{x_1}$ and $Y^{x_2}$ which depend only on the number of inspections planned. This matrix has $2^n$ rows, which correspond to all the possible combinations (paths) of success/failure of the planned inspections. The number of columns equals the number of inspections. For example, the second row of matrix $y$ $(y_2 = (110))$ corresponds to the second MAR path, where inspections 1 and 2 are successful (error-free) and inspection 3 is unsuccessful. Matrices $V^{x_1}$ and $V^{x_2}$ indicate whether MAR exceeds $R_l$ ($v_{p,k} = 1$) or not ($v_{p,k} = 0$) for each path (line) at each inspection time (column). The dimensions of vectors $W^{x_1}$ and $W^{x_2}$ are the total number of paths of their respective inspection plans. An element of these vectors equal to 1 implies that the corresponding MAR path overlaps the prohibited zone.

For the first inspection plan, there are four situations (paths) where the MAR exceeds $R_l$ ($p \times \{3, 4, 7, 8\}$), and they are denoted by $MAR^1$, $MAR^2$, $MAR^3$ and $MAR^4$. These latter correspond to the paths of the tree of the MAR (Fig. 5(a)), where there is at least one node of the MAR that exceeds $R_l$. The second inspection plan contains only three MAR paths where $R_l$ is exceeded: $MAR^5$, $MAR^6$ and $MAR^8$ (Fig. 5(b)).

For any inspection plan $x$ with $n^x = 3$, the probability of each path of the tree (regardless of whether or not it contains a MAR peak that exceeds $R_l$) is expressed as follows:

$Pr(Y^x = y_1^x) = Pr(Y^x = (111)) = (1 - \beta)^3$;
$Pr(Y^x = y_2^x) = Pr(Y^x = (110)) = \beta(1 - \beta)^2$;
$Pr(Y^x = y_3^x) = Pr(Y^x = (101)) = \beta(1 - \beta)^2$;
$Pr(Y^x = y_4^x) = Pr(Y^x = (100)) = \beta^2(1 - \beta)$;
$Pr(Y^x = y_5^x) = Pr(Y^x = (011)) = \beta^2(1 - \beta)$;
$Pr(Y^x = y_6^x) = Pr(Y^x = (010)) = \beta^2(1 - \beta)$;
$Pr(Y^x = y_7^x) = Pr(Y^x = (001)) = \beta^2(1 - \beta)$;
$Pr(Y^x = y_8^x) = Pr(Y^x = (000)) = \beta^3$.

For each inspection plan $(x_1$ and $x_2$), the probability of exceeding the threshold $R_l$ can be computed by summing the probabilities
of the paths that contain at least one MAR peak that exceeds \( R_1 \):

\[
Pr(MAR^1 \text{ overlaps } R_1) = \sum_{\beta} Pr(MAR^1; \beta \text{ overlaps } R_1) \\
= Pr(Y^* = y_1^*) + Pr(Y^* = y_2^*) + Pr(Y^* = y_3^*) + Pr(Y^* = y_4^*) \\
= \beta (1 - \beta)^2 + \beta^2 (1 - \beta) + \beta^2 (1 - \beta) + \beta^3 \\
= \beta \\
Pr(MAR^2 \text{ overlaps } R_1) = Pr(Y^* = y_1^*) + Pr(Y^* = y_2^*) + Pr(Y^* = y_3^*) \\
= \beta^2 (1 - \beta) + \beta^2 (1 - \beta) + \beta^3 \\
= 2\beta^2 - \beta^3
\]

In this example, \( Pr(MAR^1 \text{ overlaps } R_1) > Pr(MAR^2 \text{ overlaps } R_1) \) \( \forall \beta \in [0, 1] \), which means that inspection plan \( x_1 \) dominates inspection plan \( x_2 \). In fact, \( x_2 \) is one of the optimal solutions for this example, since it minimizes the probability of exposure to an MAR exceeding \( R_1 \). However, no conclusion can be drawn about the optimality of \( x_2 \) with the respect to other values of \( R_1 \). Experiments are needed to characterize and analyze the behaviour of optimal solutions with the various parameters of the model \((H, R_1, \beta \text{ and } n)\) which is our aim in the following section.

4. Experiments and discussion

The experiments were carried out using small instances generated in order to analyze the effect of type-II errors on mastery of the stochastic behaviour of MAR. The ILP was coded and solved using \textit{IBM ILOG CPLEX Optimization Studio} with an Intel Core Duo CPU 2.26GHz machine. The experiments were conducted by solving the ILP resulting from each combination of the following parameter ranges: \( H \in \{10, 20, 30, 40\} \); \( R_1 \in \{1, 2, \ldots, 9\} \) and \( \beta \in \{0.01, 0.03, 0.05, 0.07, 0.09, 0.1, 0.3, 0.5, 0.7, 0.9\} \).

4.1. Computation time

The evolution of the computation time of the optimal inspection plan is summarized in Fig. 6. It shows the variation of the average CPU time with the various values of \( \beta \) for each value of the ratio \( N/H \). For instance, when \( N/H = 0.3 \), the average value of CPU time, among the various values of \( \beta \), varies from 0.85\,s to 2.53\,s, with an overall average of 1.47\,s. The average CPU time increases slowly until it reaches its maximum at \( N/H = 0.7 \), where \( Avg_{R_1/H} \text{Avg}_{\beta} \text{CPU}(R_1/H, \beta) = 2.59 \,s \), and then drops rapidly to 1.47\,s. This is because the solution procedure explores the entire solution space, the cardinality of which grows with \( n \) as the
The probability threshold \( P_{rl} = 0.1 \) defines the combinations of \( R_l/H \) and \( N/H \) values with which the probability of exposure to a risk exceeding \( R_l \) is greater than or equal to \( P_{rl} \). The second zone corresponds to the combinations of \( R_l/H \) and \( N/H \) values with which there exists at least one inspection plan that warrants a probability of exposure to a risk exceeding \( R_l \) less than or equal to \( P_{rl} \). The first zone expands when \( \beta \) increases, at which point the second zone shrinks. This leads to fewer possibilities in the choice of \( R_l \) and/or \( N \) to guarantee a given threshold of the overall probability of risk exposure. For instance, if \( \beta = 0.05 \) and \( R_l/H = 0.2 \), \( N/H \) should be greater than or equal to 0.4, in order to be able to warrant that an optimized inspection plan with an objective function less than or equal to \( P_{rl} = 0.2 \). If \( N/H \) is less than 0.4, there is a probability of more than 0.2 of exposure to a risk of loss of more than \( R_l \) products.

4.3. Exploitation of the results

These experiments show that type-II inspection error has an impact on the optimal inspection plan that minimizes the probability of risk exposure from an insurance perspective. Although the tested instance is relatively short, the experiments provide some insights into how to design risk-based inspection plans when inspection resources are not error-free. To illustrate, let us take an example showing how these results can be exploited. The following steps are required:

1. Specify the framework and the input parameters: subsystem and failures to be monitored, quantities of products to be manufactured during the planning horizon, and inspection resource effectiveness.

2. Knowing the value of \( \beta \), draw a corresponding sub-plot similar to those in Fig. 8. \( \beta \) should reflect the ability of the inspection resource to detect the occurrence of a failure event. In fact, type-II errors can be caused by any or all of the following: measuring tool ineffectiveness (gauge capability), failure diagnosability, sampling strategy (size).

3. Knowing the threshold of risk exposure \( R_l \), draw a vertical line corresponding to \( (R_l/H)_{target} \). As shown in the example in Fig. 9 for \( \beta = 0.07 \), the intersection points of this line with the various contours of objective function levels make it possible to determine the minimum values of \( N/H \) to warrant each of them. For instance, with \( N/H \geq (N/H)_{min} \), there is an inspection plan that has a probability that is less than or equal to 0.3 of leading to an MAR that exceeds the threshold \( R_l \) that corresponds to \( (R_l/H)_{target} \). Similarly, this probability is greater than or equal to 0.9 when \( N/H \leq (N/H)_{min} \).

4. Knowing the maximum inspection capacity, draw a horizontal line corresponding to \( (N/H)_{target} \). As shown in Fig. 9, the intersection points of this line with the various contours of the objective function levels make it possible to determine the minimum values of \( R_l/H \) that could be achieved with a given probability, by an optimized inspection plan. For instance, with \( (N/H)_{target} = 0.2 \), the optimized inspection plan has a probability that is less than or equal to 0.9 of leading to an MAR that exceeds 0.31 of \( H \), a probability that is less than or equal to 0.2 of leading to an MAR that exceeds 0.39 of \( H \) and a probability that is less than or equal 0.1 to leading to an MAR that exceeds 0.46 of \( H \).

These experiments enabled us to analyze the effect of type-II error on the optimized inspection plan from an insurance perspective. A kind of guideline is derived to explain how similar results could be exploited to take into account the inspection error when designing an inspection plan with massive loss risk awareness.
Fig. 7. Optimal solution surfaces for different values of $\beta$. (For interpretation of the references to color in the text, the reader is referred to the web version of this article.)

Fig. 8. Contours of the optimal objective function: $R_l/H$ vs $N/H$. 
5. Conclusion

This paper studies the effect of type-II inspection error on the effectiveness of an inspection plan from an insurance viewpoint. To do so, we have introduced the probability of type-II error in the model of MAR, and used it to control risk exposure. A linear programming formulation, including the stochastic behaviour of the MAR, has been proposed and solved. Experiments have been conducted to analyze the effect of inspection error on risk exposure control.

The impact of type-II inspection error on the material-at-risk control is found to be significant. It was expected that the greater the risk of non-detection is, the greater the number of inspections must be increased to ensure a given probability of falling in the prohibited zone of over-exposure. However, the merit of this document is to provide an advance in Risk Exposure Control Approach based Quality Inspection Plan which is adjusted according to the degree of confidence required when considering type-II inspection error.

The perspectives of this work are basically related to finding efficient algorithms (exact or heuristics) in order to cope with the combinatorial explosion of the problem with large instances. Moreover, the model could be generalized when extended to a production system where several production resources subject to failure are monitored by product inspection plans with limited inspection and correction resources.

References


