A systemic approach of quality controls

S. Bassetto, C. Paredes, B. Baud-Lavigne
Mathematics and Industrial Engineering
Polytechnique Montréal
2500, Chemin de Polytechnique
Montréal, Québec, Canada

Abstract—The article presents a systemic approach of quality control that takes into accounts various uses of data issued from controls charts. The article proposes an enhancement of economic design of quality control chart, using learning as an inner parameter of the design. By doing so, the economic design of control chart becomes dynamic. The method employed in this research is based on the enhancement of the model of Baud-Lavigne, Bassetto and Penz and a simulation to test the model. The result is a dynamic model of the design of control charts that links learning and number of controls. First simulations retrieve a slight decrease of quantity of controls while yield improvement occurs.

Keywords—dynamic quality control, economic design of control

I. INTRODUCTION

Quality controls provide data that are reused throughout the production system. Several teams are involved to define an adequate level of data collection. These experts report to several organizations. Internally they can be focused on quality, production, process control, maintenance, yield or product. Externally, they can be suppliers or customers. An extended organization reuses then quality measurements to control tools health, processes drifts, products quality, but also to qualify suppliers, investigates problems, increase yield and report to customers. Changing the frequency of controls or number of data collected is often an overwhelming task that requires a strong lobby activity. Unfortunately, this systemic view of quality controls is hardly translated into models that are employed to design quality controls. As a consequence models of control charts are often designed in a silo-mode focused on a process or a specific failure mode, while in the mean time associated data are actually reused for many purposes. This leads at sub-optimal poorly effective data collections.

The research presented in this paper helps to cross these borders. It provides a more systemic view of quality control model. The key idea modelled is that SPC data [1] [2] are reused for yield improvement, scrap investigations and reduction. The research provides an economic model of statistical control charts that enhances the one presented by Baud-Lavigne, Bassetto and Penz [3]. In [3] scrap rate is given, while in our research it is influenced by the quantity of controls. The proposal of this research is to provide managers a tool to adequately adjust quality control plan accordingly to scrap rate.

Wright has theorized first learning in production by linking production volume to cost improvement [9]. Learning phenomenon has been intensively studied since.

Tapiero has been pioneered with his article linking production learning and quality improvement [10]. Fine has opened also a new perspective by adapting quality controls, relatively to improvement actions and learning. He proposes an integrated model of quality control with maintenance, by including the learning of the operators of the production process [4]. Operators can discover and eliminate defects if, during an inspection, they find that the process is out of control. As a consequence, quality controls can be adapted accordingly to these improvements.

Close to authors concern is the work of Yang, Wang and Pai. They have identified a link between the statistical process control and the learning effect on the standard deviation of the process. This learning is caused by the improvement of the quality program. They propose a modification of X-control charts by considering learning [6].

Lapré, Mukherjee and Van Wassenhove [9] worked in the understanding of learning mechanism. They have presented the scrap-rate function as a function of learning, in two parts: an autonomous part and an induced part. These two sides are affected by the quantity of goods produced. The autonomous part is a natural learning obtained from experience and the induced part that is an accelerated learning driven by an agent, for instance an improvement of know-how of the production staff. The induced part is also divided into aspects: conceptual learning and operational learning. The conceptual one is the process of acquiring a better understanding of cause and effect relationships, also known as know-why. The operational one is the process of obtaining validation of the action-outcome relationships also designated as know-how.

Baud-Lavigne, Bassetto and Penz [3] have extended the model of Lorenzen and Vance [7] of design economics of control charts, studied first by Duncan [8]. They add a link between SPC control plan (sampling rate and number of control) and the duration of scrap investigation. As illustrated figure 1, the scrap rate was considered as an exogenous parameter of the model. They issued an economical model of quality control charts that
includes scrap analysis and improvement actions as part of the equation. Their model has been the starter of the present work.

Figure 1. Model of the production line and the quality controls by [3].

The mastery of the production system is then directly linked at quantity of good produced and at the quality policy. As stated in [3], data issued from controls are enablers of improvement actions. These actions should normally reduce scraps. Moreover, controls during the production, helps in monitoring processes, reducing also scraps. Then scraps are no more external parameters, they are influenced by the quantity of control, as illustrated in figure 2.

Figure 2. Relationship between learning, production and quality control.

The objective of this paper is to provide a control chart model that identifies the optimal amount of controls in a context of interaction between production, quality controls and learning. This paper is structured around two parts. Section 2 presents the model. It details how the scrap rate is included as endogenous function of learning. The section 3 presents the test and associated discussions.

II. THE PROPOSED MODEL

The model proposed is based on an economic evaluation of controls. Central to this model is the cost function expression. Noted C, it is influenced by the scrap rate S, as presented by [3]. However in this model, the scrap rate is an exogenous parameter, meaning that S was constant over the simulation. In actual operations, online monitoring and learning effects influence the scrap rate. In order to include the scrap rate S as endogenous parameters of the cost function, the model of [8] has been considered. The general scrap function defined by [8] is as follows:

\[
S(t) = \exp \left( a + \mu Z(t) \right)
\]

(26)

where a is the initial scrap rate of the production process, \( \mu \) is learning rate and \( Z(t) \) is the quantity of products produced since the first period of production until t.

The extension of the equation (26) has included three factors and two assumptions. The first factor is the autonomous part of the learning. It depends on the cumulative quantity of goods produced cumulative so far [9]. The second one is the induced part. It depends of the cumulative amount of controls per hour. The third factor affects both autonomous and induced parts and is a forgetting factor as presented by [11].

The assumptions are made based on authors' industrial experience. In many enterprises the monitoring of scrap rate, as yield is performed on a regular basis: the week or the month. This leads at the first assumption.

\textbf{Assumption 1:} The learning from on-line monitoring is monitored sequentially and regularly.

The major consequence of Assumption 1 is that the evolution of scrap rate is represented sequentially. For a given amount of observations – or productions – the scrap rate evolves by steps. This represents a classical management case in industry, where yield and scrap rates are revised weekly if not monthly. Over a given period of time (indexed by j), the scrap rate evolves from a starting value, noted \( S_0(j) \) to a final value \( S_f(j) \). This assumption enables also the simulation of the dynamic induce by embedding S as an endogenous parameter. The manner \( S_0(j) \) moves to \( S_f(j) \) is out of the scope of this study.

\textbf{Assumption 2:} The scrap function has been formulated in a simplified model of one tool and one measurement device [1]. This hypothesis has been adopted here in order to build a first tractable model.

\textbf{Assumption 3:} The learning rate for autonomous part and induced part are constant over time and production. This assumption presents the case where the manufacturing organisation is stable during the considered period.

Notations:
- \( j \): the index of current period \( j \in [1: N_{\text{max}}] \)
- \( N_{\text{max}} \): the duration of monitoring
- \( S_0(j) \): the initial scrap rate of the production process at step j
- \( S_f(j) \): the final scrap rate of the production process at step j
- \( \mu_a \): the learning rate of the autonomous part
- \( \mu_i \): the learning rate of the induced part
- \( Z_k \): the quantity of products produced during the period k
• $n$: the number of measurement. It is assumed that one measurement provides one data

• $h$: the interval between measurements, expressed in hours

• $\left(\frac{n}{h}\right)_k$: the quantity of control done during the period $k$

• $factor_{k}^{at}$: the forgetting factor of autonomous learning, at the stage $k$

• $factor_{k}^{in}$: the forgetting factor of induced learning, at the stage $k$

• The algorithm of [3] is noted BLBP

The computation of the scrap function in the simulation is extended expression (26) in the following:

$$S_f(j) = \exp \left( \frac{1}{\sum_{k=1}^{j} \text{factor}_{k}^{at} \times \left( \frac{n}{h} \right)_k} + \frac{1}{\sum_{k=1}^{j} \text{factor}_{k}^{in} \times \left( \frac{n}{h} \right)_k} \right)$$

(27)

The computation of the scrap function in the simulation is noted in the remainder $CB$.

The implementation has been developed under Scilab [12]. Same values have been employed than in [2]. It has been assumed that process drifts to 1 sigma every 24 hours in average. Costs are estimated upon a cost of a machine of 300 $/h. Quality cost/hour while in-control is roughly estimated at 160 $/h. Quality cost/hour while out-of-control reaches 640 $/h. Sampling parameters considers a fixed cost per sample of 2 $ (a 1000$ test wafer is used about 500 times), a time to sample 17 units takes about 5 minutes i.e. $5.10^{-3}$ min/unit sampled. This leads to a variable cost of 1.4706$ (time to sample x 300$/h). The search time following an out of control is one hour, whenever the process is in control or not. So the cost of a false alarm is 300 $ (1 hour x 300$/h). The time to repair the process is 4 hours, and the cost to locate and repair the assignable causes is 2000$ (5 hours x 300$/h). The maximum gain over one scrap analysis is estimated at 2500$. The scrap rate for this simulation starts at 50%. The manufacture processes 12 products per hour. The autonomous learning is considered as permanent. The induced learning is considered to have 3 months validity. Each simulation period $j$ is considered as one month of production.

$$\forall k \in [1 : N_{\text{max}}], \text{factor}_{k}^{\text{in}} = 1$$

$$N_{\text{max}} = 6. \text{ This simulates a six-month production.}$$

<table>
<thead>
<tr>
<th>Period j</th>
<th>$n/h$ (controls per hour)</th>
<th>Scrap rate per period</th>
<th>Cost Period per</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.084</td>
<td>0.489</td>
<td>1 2176,02</td>
</tr>
<tr>
<td>2</td>
<td>8.079</td>
<td>0.4329</td>
<td>9 463,43</td>
</tr>
<tr>
<td>3</td>
<td>8.059</td>
<td>0.2997</td>
<td>7 589,37</td>
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<td>4</td>
<td>8.050</td>
<td>0.2619</td>
<td>5 953,33</td>
</tr>
<tr>
<td>5</td>
<td>8.047</td>
<td>0.2495</td>
<td>4 395,82</td>
</tr>
<tr>
<td>6</td>
<td>8.046</td>
<td>0.2490</td>
<td>2 840,72</td>
</tr>
</tbody>
</table>

Table 1. Results

The initial period the quantity of control is 8.084 controls per hour, while in the final period this rate drop down to 8.046. This represents an evolution of $\frac{0.004-0.004}{0.004} \times 100 = 0.47%$. In the mean time, the scrap rate evolves also. The initial scrap rate is 50% and it decreases down to 24,90% after six months of continuous improvement and learning. This represents an evolution of $\frac{0.5-0.2490}{0.5} \times 100 = 50,2%$. 

III. SIMULATION AND DISCUSSION

The central idea to compute the evolution is to apply the algorithm BLBP then the equation $CB$, recursively. The general process is illustrated Figure 3. For a given period $j$, the amount of data is considered as optimal and can be noted $\left(\frac{n}{h}\right)_j$. It is computed by the algorithm BLBP. After, $CB$ computes the final value of scrap for the period $j$. This process applied $N_{\text{max}}$ times transforms the scrap rate from $S_0(1)$ to $S_f(N_{\text{max}})$.

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Results presented Table 1 are also drawn figure 4. This graphic is in two parts. Above is the number of controls per hour, evolution. The second part represents the scrap rate per period. Globally the two curves decreases and follow the same patterns. There is no time delay between the two curves. It can also be noticed that the second point for each of these curves is slightly shifted from the natural curve fitting. This is confirmed by data as the improvement between the period 1 and period 2 is lower than the others.

The reduction of number of controls per hours between period 1 and period 2 is 0.06. Between period 2 and period 3, the reduction equals 0.24%, between period 3 and 4, controls are reduced by 0.11% the 0.03% and finally between period 5 and period 6, they are decreased by a factor of 0.012%.

The scrap reduction between the period 1 and 2 is 11.4%. Between period 2 and 3, the reduction is the highest at 30.76%. Between period 3 and 4 scrap rate decreases of 12.61%. Between period 4 and 5, it decreases of 4.73. And the transition between the two last periods sees the scrap rate reducing of 0.2%.

The first observation that can be noticed on this dynamic system is its stability. It has not shown instability for instance: less controls, more scraps, more control less scraps…. This comes from the model behind that cumulates learning. It is not proven that other configurations, especially for other values of forgetting factors would not lead at an instable model.

The second observation is a difference with [3]. Now, the scrap rate and the quantity of control are interdependent. In [3] scrap rate was a given parameter, and was not influenced by the number of controls. As scrap rate and number of controls decrease repeatedly, period after period, the cost also reduces.

The third observation is the weak decrease of number of controls. Contrary to [3], where a high variation of scrap rate, generate a high variation of optimal control, the proposed algorithm does not reproduce such a variation. This result is counter-intuitive and remains to be deepened.

Finally, it can be noticed that the scrap rate is very sensitive at small control per hour changes while in the mean time the quantity of controls are less sensitive to scrap reduction. There is then an asymmetrical phenomenon observed that should be investigated in deep in further researches. Considering scrap rate as an endogenous parameter generates an internal dynamic to controls that can be far more complicated than expected.

IV. CONCLUSION

This project provides an evolution of design economic of X-control charts provided by [3], in order to include scrap-rate as an endogenous parameter. The model provides an integrated analysis of quantity of controls, production and learning. The scrap rate evolution is modeled as a function of quantity of productions produced so far and the quantity of information earned on products through controls. These learning are weighted by forgetting factors. The paper tests also the dynamic of the proposed model on a reference case study. A recursive algorithm has been settled that computes alternatively the scrap improvements due to measurement and learning then the new value of optimal measurements. The dynamic system tested does not present any instability. However peculiar counter intuitive results have been pointed out. The quantity of controls is almost stable, while the quantity of scraps is very variable. The dynamic of this system has then to be systematically investigated as the fine understanding of why these counterintuitive events happen.

V. AUTHORS CONTRIBUTIONS AND ACKNOWLEDGEMENT

SB led the project and wrote the article. CP developed algorithms, simulations. BBL initiated the program. This project has been sponsored by NSERC discovery grants No418674.

REFERENCES


