Module Selection and Supply Chain Optimization for Customized Product Families Using Redundancy and Standardization

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Abstract—In this paper, we study the problem of configuring a product family which has to satisfy diversified customer requirements. Modular design strategies allow a bill of materials to be generated for various finished products from a limited subset of modules. Simultaneously, a production location must be selected for the manufacture of each module. From a product point-of-view, the strategies adopted are often extreme, proposing either the manufacturing of the total diversity (all possible products) or a limited set of standardized products (only a few products are proposed). The objective of this paper is to investigate intermediate cases on this continuum, in order to better understand the potential for profit. Our comparison is based on the product family configuration (selection of the best modules) that minimizes production and transportation costs under time constraints.

Note to Practitioners—In the context of mass customization, it is often difficult to evaluate advantage/disadvantage of personalization versus standardization. This paper provides answers to this critical question considering intermediate cases (different levels of standardization and different levels of redundancy). We consider the selection of modules for a product family, minimizing production and transportation costs. Intermediate cases show that partial standardization of the product family leads to large benefits on the supply chain while redundancy has small impact. We can summarize our results in a few succinct points: from an economic point-of-view, the standardization strategy leads to greater benefits than redundancy; the redundancy strategy is practically not interesting except when production costs are greater than assembly costs; the standardization strategy is much more profitable mainly when fixed costs are greater than variable costs; total standardization leads generally to offer very few different products except when variable costs are greater than fixed costs; when variables costs become such higher that we can ignore fixed costs, then the basic model, the standardization and the redundancy strategies, provides the same optimal solution.

Index Terms—Configuration, modular design, optimization, product family, supply chain.

I. INTRODUCTION

FOR SEVERAL decades now, the industry has attempted to propose increasingly diversified products in order to satisfy most segments of the market [25]. With this marketing approach, the design of unique products that are independent of one another is dropped in favour of families of products, based on a common platform, with a set of options which makes it possible to achieve the desired diversity [24].

This has resulted in the evolution of product design, to the point where a finished product is now seen as a base to which options are added which permit customization. Mass customization is the use of flexible manufacturing systems to produce diversified products with a mass production output. From a production point-of-view, this diversity (which represents the customization level attributed to a product family in order to respond to the customer specifications) is difficult to manage. If manufacturing the base leads to the implementation of many similar production lines, then finalization of the product becomes very complex. Furthermore, the product very often cannot be manufactured in a short time as required by the customer, or at what the customer considers a reasonable price [19]. Then, the challenge is to build rapidly at a low price a customized product. Several strategies were proposed in the literature. Among them, the assemble-to-order strategy consists of building the product from parts manufactured in advance. These parts, also called modules, are often made to stock. The grouping of functions into modules assembled in advance makes it possible to resolve these difficulties, since these modules, in smaller numbers, can be made for stock in low-cost production facilities, and sent to a final assembly site close to the market. This is the concept of modular design. The present study follows this assumption.

The objective of this study is to find an acceptable balance between the number of modules to manufacture at distant sites, to choose those sites according to manufacturing and logistical costs, and to determine the bills of materials of finished products to satisfy the time constraints of the final assembly. For the most part, two extreme strategies exist to achieve this. The first is to define a limited number of modules which will serve as a base on which to create the bill of materials for every product (possibly including one or more functions in the product, even if the customer does not ask for them). This is the principle of standardization, which makes it possible to reduce the logistical
costs generated by diversity, even if it means losing money on some finished products. The second strategy is to create bills of materials corresponding exactly to the required composition of the finished product (and no extra functions). In that case, the profits on the cost of components are obvious, but the cost of the management of the diversity increases.

In our investigation of intermediate strategies, we consider standardization (in this case a limited number of not required functions may be included to any product) and redundancy (here the same function may appear independently in different modules of the same product). To our knowledge, there is no work on these strategies in the literature. We first give an outline of the work that does exist in the literature (Section II). The problem is described formally in Section III, and the proposed models are presented in Section IV. The experiments are presented in Section V. We conclude and suggest some future research tracks in Section VI.

II. STATE-OF-THE-ART

Mass customization, which is aimed at meeting the needs of individual customers, while ensuring the low costs and high level of responsiveness typically achieved by mass production [3], has received extensive attention since its emergence. Manufacturers must differentiate their products by focusing on individual customer needs without sacrificing efficiency, effectiveness, and the low cost customers expect.

The challenge of designing product families with a common platform in order to achieve product customization, while maintaining the economy of scale of mass production, has been well recognized in academia and industry alike [37].

Delayed differentiation or Postponement is a widely used concept in product families. The manufacturing process starts by making a generic product structure that is later differentiated into specific finished products. Feitzinger and Lee [12] explain that the key of mass-customizing is postponing the task of differentiating a product for a specific customer until the latest possible point in the supply network. They identify three organizational-design principles which form the basic building blocks of an effective mass-customization program. They discuss the case of the Hewlett-Packard Company. Garg and Tang [16] develop two models to study products with more than one point of differentiation. In each model, they examine the benefits of delayed differentiation at each of these points, and derive the necessary conditions when one type of delayed differentiation is more beneficial than the other. Their analysis indicates that demand variabilities, correlations and the relative magnitudes of the lead times play an important role in determining which point of differentiation should be delayed. Yadav et al. [40] formulate a multi-objective problem to select a product family and design its supply chain in order to maintain product differentiation and help trade-off the cost and price premium drawing capability. They use an Interactive Particle Swarm Optimization (IPSO) approach. A case study for a wiring harness supplier of an Automated Guided Vehicle manufacturer is considered and IPSO is implemented to solve it.

Integrating modules of components into the design is a strategy that helps customize a large variety of high-demand products. Modularization makes it possible to organize complex designs and process operations more efficiently by decomposing complex systems into simpler portions [24], [28], [34]. A module can be defined as a group of standard and interchangeable components [14]; it is a complex group that allocates a function to the product and which can be changed, replaced, and produced independently [39]. A modular system is made up of independent units which can be easily assembled and which behave in a certain way in a whole system [5]. The term modularity is used to designate a common and independent part for the creation of a variety of products [21].

Lee and Tang [27] develop a simple model that captures the costs and benefits associated with the redesign strategy (which consists to delay the point of product differentiation). Then, they apply this simple model to analyze some special cases that are motivated by real examples. These cases enable them to formalize three different product/process redesign approaches (standardization, modular design, and process restructuring) for delaying product differentiation that some companies are beginning to pursue. Fujita [13] discusses the product variety design under an optimization viewpoint, he uses optimization to determine the contents of modules and their combinations under fixed modular architecture. He draws three classes of optimization problems: module attribute assignment, modules combination and the simultaneous design of both. Then, he gives two typical examples through aircraft design for optimal attribute design and television receiver design for optimal module combination. Swaminathan and Tayur [36] model the problem of finding the optimal configuration of inventory levels of semi-finished products (modules) that can serve more than one final product in a stochastic demand environment as a two-stage integer program. They use structural decomposition of the problem and subgradient derivative methods to provide an effective solution procedure. Qian and Konz [30] investigate the effects of component commonality and various price-demand functions on the supply chain performance in profit. The benefit with component commonality is illustrated with an application case involving a product family of cordless drills. Results show that optimal profits and sale prices of the product family could change significantly with a change of the price-dependent demand function.

At the same time, the concept of supply chain management is garnering a great deal of interest, since the opportunity for integrated supply chain management can reduce the propagation of undesirable (or unexpected) events through the network, and can decisively affect the profitability of all its members [18]. There have been several articles recently on modeling traditional supply chain management, which can be classified into two major categories: configuration-level issues and coordination-level issues [19]. The configuration-level issues include articles on the following:

- Product design decisions, which deal with product types, materials to be used, product differentiation, and modularity [9], [17].
- Supply decisions, which are aimed at determining the supply strategy (make or buy decisions, outsourcing, among others), and also at determining which suppliers have to be selected [8], [23], [32].
The producer has only a short time \( T \) in which to respond to each customer demand. That time is less than the time required to assemble the products from elementary components. In the basic problem, the producer has to provide the product exactly according to the customer’s requirements (without extra features). This constraint comes from technical considerations or simply to avoid the supplementary cost of providing features that were not requested. In this paper, we explore models that relax these strong assumptions.

To satisfy customers, the producer brings in preassembled components, called modules, from many suppliers located at distant facilities around the world. The suppliers’ facilities are characterized by low production costs. The modules are then assembled at the producer’s facility, which we assume to be close to the customers and thus characterized by a high level of responsiveness and reduced lead time.

This problem occurs in the automotive industry, and more specifically for the manufacturing of electrical beams. Electrical beam is one of the first parts assembled in a car, just after the vehicle is entered in the assembly line. The time needed to build the requested beam from scratch is larger than the time available. Then, the electrical beam must be built from subparts called modules. The challenge for these companies is to decide the set of modules that have to be produced in order to respect the time delay for all possible orders. As the modules are made to stock, the strategy used is to produce the modules in low cost countries, and transport them in the nearby assembly facilities [25].

The strategic problem is then to configure the product family, i.e., to determine the bill of materials for each product. A product will be made up of a set of modules. Simultaneously, for a set of required modules, i.e., the modules that appear in at least one bill of materials, we determine where those modules must be produced in order to minimize production and transportation costs. The model considered here is proposed to take strategic decisions (modules selected for the bill of materials and production sites) for a long term period, and not to solve planning issues. Consequently, all the costs are estimations of the total costs during the time horizon of the application of the decisions. For example, the transportation costs are estimated means considering that the transportation policy is fixed. At this level, tactical concerns as transportation or storage capacity, inventory management, lead time for example are not taken into account.

The various elements of the problem, as well as the main costs to be taken into account, are described more formally below. First, we introduce the notions of functions, products, modules, and distant sites.

- \( \mathcal{F} = \{F_1, \ldots, F_q\} \): set of \( q \) functions that can appear in both finished products and modules. A function is a product feature that corresponds to a customer requirement. We suppose here that all functions are independent.
- \( \mathcal{P} = \{P_1, \ldots, P_n\} \): set of \( n \) possible finished products that may be demanded by at least one customer, with \( D_t \) the estimated demand of the product \( P_t \) during the life cycle of the product family.
- \( \mathcal{M} = \{M_1, \ldots, M_m\} \): set of \( m \) possible modules.
- \( \mathcal{S} = \{S_1, \ldots, S_s\} \): set of \( s \) distant production facilities

The problem data are expressed as follows.


- $F^A_j$: the fixed cost of module $M_j$ at the nearby facility (management costs).
- $V^A_j$: the variable cost of module $M_j$ at the nearby facility (assembly, storage, transportation, etc.).
- $F^P_{ij}$: the fixed cost of module $M_j$ at facility $S_i$ (management costs).
- $V^P_{ij}$: the variable cost of module $M_j$ at facility $S_i$ (assembly, storage, etc.).
- $t_j$: the time required to assemble module $M_j$ into a finished product.
- $T$: maximum assembly time allowed for a finished product.
- $W_{ij}$: the workload caused by producing module $M_j$ at facility $S_i$.

Fixed costs $F^A_j$ and $F^P_{ij}$ include all the costs related to the implementation of the production in a site, buildings, machines, tools, a part of human resources, and the information system for example. It may also contain recurrent costs that are independent of the production quantities, as insurance, maintenance costs, etc. Variable costs $V^A_j$ and $V^P_{ij}$ concern all the costs that are directly dependent on the production quantities as energy, the row materials, the transportation, and a part of human resources for example.

Under these assumptions, a product (or a module) is represented by a binary vector of size $q$. Each element shows whether the corresponding function is required in the product ($\forall i \in \{1, \cdots, n\}$) or not ($\forall i \in \{1, \cdots, q\}$). A function is entirely assured by a module, and functions and/or modules are independent. The selection of a module in a bill of materials does not imply the selection (or the rejection) of another one. Then, inclusion or exclusion are not taken into account. We assume that a function is entirely furnished by a module (a 1 in the vector signifies that the function is in entirely). The set $\mathcal{M}$ contains $m$ modules. $\mathcal{M}$ may be all the possible modules from the whole combinatory or a subset of those modules defined by the engineering.

The problem of optimization is now simple to express. It is necessary to determine the subset $\mathcal{M}'$ of modules that has to be manufactured. This subset has to contain all the modules necessary for the elaboration of the bills of materials of all the possible finished products. When all the bills of materials (Fig. 2) have been determined, we can easily deduce the demand for each module and assign its production to the various distant production sites. As we have just stated, a natural initiative could be to first determine the subset $\mathcal{M}'$ and define the bills of materials of finished products, and then to assign the production of modules to those distant sites. Our results show that this approach is not successful on problems where there is no standardization [10]. The objective is to solve this optimization problem globally, rather than to undertake a succession of partial optimizations.

The bills of materials shown in Fig. 2 correspond to the assembly strategy of producing a finished product exactly as needed, i.e., without extra functions (if function $k$, for example, is not present in the product, then it must not be present in the modules constituting that product’s bill of materials), and without function redundancy (if function $k$ is present in the product, then it is present in only one module among those constituting that product’s bill of materials). Other assembly strategies are explored in this paper as well, like the standardization strategy (authorization to include extra functions in the finished product that were not requested) and the redundancy strategy (the same function could be present in more than one module in the product’s bill of materials).

The described problem is NP-hard in the strong sense, because it includes the classic set partitioning problem [15].

IV. MATHEMATICAL MODELS

First, we present the optimization model, which allows us to determine optimal solutions for the problem of total diversity, i.e., without standardization or redundancy. Then, we define the model in which we accept a limited number of supplementary functions, but without function redundancy, i.e., the same function is not present in more than one module. Finally, we present the model with extra functions and function redundancy.

A. Model Without Standardization or Redundancy

In order to solve this model optimally, it will be necessary to precisely determine the bill of materials for each product. For this, we define the binary variable $X_{ij}$, which takes the value 1 only if the product $P_k$ has the module $M_j$ as a component. If that is the case, the binary variable $Y_j$, which means that $M_j$ is manufactured at one distant site at least $S_i$ will take the value 1. The binary variable $Y_{ij}$ takes the value 1 only if $M_j$ is manufactured, at least partially, at site $S_i$. Finally, the integer variable $Q_{ij}$ represents the quantity of $M_j$ produced at site $S_i$.

In order to simplify the writing of the model, we introduce the parameters $\delta_{ik}$ and $\lambda_{jk}$. The binary parameter $\delta_{ik} = 1$ if the function $F^A_k$ is present in the product $P_i$. Also, the parameter $\lambda_{jk} = 1$ if the function $F^P_{ik}$ is present in the module $M_j$. With these notations, we can now write the Mixed Integer Linear Program of the model. The objective function is expressed as the sum of costs

$$\min \sum_{j=1}^{m} F^A_j X_{ij} + \sum_{j=1}^{m} V^A_j \left( \sum_{i=1}^{n} D_i X_{ij} \right) + \sum_{l=1}^{s} \sum_{j=1}^{m} F^P_{il} Y_{jl} + \sum_{l=1}^{s} \sum_{j=1}^{m} V^P_{ij} Q_{jl}$$

s.t.

$$\sum_{j=1}^{m} \lambda_{jk} X_{ij} = \delta_{ik} \quad \forall i \in \{1, \cdots, n\}, \forall k \in \{1, \cdots, q\}$$

$$\sum_{j=1}^{m} t_j X_{ij} \leq T \quad \forall i \in \{1, \cdots, n\}$$

$$X_{ij} \leq Y_j \quad \forall i \in \{1, \cdots, n\}, \forall j \in \{1, \cdots, m\}$$

$$\sum_{j=1}^{m} Q_{jl} = D_i X_{ij} \quad \forall j \in \{1, \cdots, m\}$$

$$\sum_{j=1}^{m} W_{ij} Q_{jl} \leq W_l \quad \forall l \in \{1, \cdots, s\}$$

Fig. 2. Which modules must be included in the bills of materials?
\[ Q_{j} \leq K_{j}Y_{j} \quad \forall j \in \{1, \ldots, m\} \forall t \in \{1, \ldots, s\} \]  \hspace{1cm} (7)

\[ Q_{j} \geq 0 \quad \forall j \in \{1, \ldots, m\} \forall t \in \{1, \ldots, s\} \]  \hspace{1cm} (8)

\[ Y_{j}, Y_{j} \in \{0, 1\} \quad \forall j \in \{1, \ldots, m\} \forall t \in \{1, \ldots, s\} \]  \hspace{1cm} (9)

\[ X_{ij} \in \{0, 1\} \quad \forall i \in \{1, \ldots, n\} \forall j \in \{1, \ldots, m\} \]. \hspace{1cm} (10)

The objective function minimizes the costs (fixed and variable) incurred at the nearby facility, where \((\sum_{i=1}^{n} D_{it}X_{ij})\) is the total demand for module \(M_{j}\), and the costs (fixed and variable) incurred at the distant facilities. Constraint (2) shows that a finished product \(P_{i}\) must be assembled exactly as requested by the customer. If the function is not present, then it must not appear in any of the product’s components. Constraint (3) indicates that products must be assembled within the time window \(T_{t}\) in order to respect the delivery time. According to constraint (4), if module \(M_{j}\) is used in the bill of materials of product \(P_{t}\), then the module \(M_{j}\) must be produced somewhere. Constraint (5) indicates that the production of a module \(M_{j}\) must satisfy the requirements. Constraint (6) shows that production at facility \(S_{t}\) must not exceed its capacity. Constraint (7) expresses the relation between the variables \(Q_{j}\) and \(Y_{j}\). A module \(M_{j}\) can be produced in \(S_{t}\) only if \(M_{j}\) is assigned at \(S_{t}\) (\(Y_{j} = 1\)). The parameter \(K_{j}\) is a large constant value representing the maximal quantity that can be manufactured at the distant site \(S_{t}\). It can be calculated by the following formula:

\[ K_{j} = \min \left\{ \frac{W_{t}}{W_{j}b} \sum_{i=1}^{n} D_{it} \right\} \quad \forall j \in \{1, \ldots, m\} \]  \hspace{1cm} (11)

Finally, constraints (8), (9), and (10) guarantee the positivity of the quantities of modules produced and ensure that the decision variables are binary.

**B. Model With Total Standardization and Without Redundancy**

We can now transform the previous model in order to formulate the total standardization problem. For this, we substitute constraint (2) by

\[ \sum_{j=1}^{m} \lambda_{jk}X_{ij} = 1 \quad \forall P_{t} \quad \text{and} \quad k/\beta_{ik} = 1 \]  \hspace{1cm} (12)

\[ \sum_{j=1}^{m} \lambda_{jk}X_{ij} \leq 1 \quad \forall P_{t} \quad \text{and} \quad k/\beta_{ik} = 0. \]  \hspace{1cm} (13)

Constraint (12) expresses the fact that, if a function \(F_{k}\) is present in a finished product, it must appear in precisely one, and only one, module among those detailed in the product’s bill of materials. Constraint (13) indicates that, if a function \(F_{k}\) is not present in the product \(P_{t}\), then it could appear in its bill of materials (again, only in one component).

This model thus demands that a required function be present in a single copy of the product’s bill of materials, and that a function which is not needed be present in at most a single copy of the product’s bill of materials. Later, we will investigate the variants in which the standardization is limited.

**C. Partial Standardization Without Redundancy**

Depending on the industrial strategy adopted by the company, standardization can be limited. Limitation may occur from a desire to reduce the cost generated by adding unneeded functions to the product, or by other objectives, such as not increasing the weight of the product for example. For example, it can be detrimental to install an electronic card in a laptop if the card is not necessary (supplementary weight, more energy consumption ...).

In that case, a constraint must be added to the model Section IV-B. in order to limit standardization. This constraint is expressed as follows:

\[ \sum_{j=1}^{q} \sum_{j=1}^{m} \lambda_{jk}X_{ij} \leq f_{i} + \alpha_{i} \quad \forall P_{t}. \]  \hspace{1cm} (14)

Constraint (14) makes it possible to count the total number of functions provided by the product’s components (bill of materials). The value \(f_{i}\) gives the number of functions needed in \(P_{t}\) and the parameter \(\alpha_{i}\) is the number of extra functions tolerated for the product. We note here that it is possible to limit standardization in a specific way for each finished product.

**D. Model With Function Redundancy and Without Standardization**

In certain applications, redundancy is sometimes acceptable. It occurs when a requested function is installed twice (provided by two different modules) in the same finished product. This is common, in the computer industry. Let us suppose that a manufacturer proposes two versions of a computer. The first version contains, among other things, a motherboard and a basic graphics card. The more sophisticated version has the same motherboard, but a more powerful version of the graphics card, requested by only 5% of customers who are interested in video games. The manufacturer can assemble the motherboard and the appropriate video card according to the customer’s request, but he can also install a motherboard in this computer which already contains the basic graphics card, and install it in all the computers. He will add to this motherboard, which includes the basic graphics card, the more powerful graphics card when asked to do so by the customer. He will then have only two cards to manage, and for only 5% of the customers will he have an over cost on the components of the basic graphics card. In that case, the same function, \(F_{k}\), should appear several times in the product, and supplementary constraints should be added to avoid the appearance of that function several times in the product’s bill of materials.

The redundancy that we address here is different and concerns the apparition of the same function, \(F_{k}\), several times. This is the case, for example, with the electric beams, where there can be wires in a beam that are not used. In our modelling, it is sufficient to replace constraints (12) and (13) by the following ones:

\[ \sum_{j=1}^{m} \lambda_{jk}X_{ij} \leq 2 \quad \forall P_{t} \quad \text{and} \quad k/\beta_{ik} = 1 \]  \hspace{1cm} (15)

\[ \sum_{j=1}^{m} \lambda_{jk}X_{ij} \geq 1 \quad \forall P_{t} \quad \text{and} \quad k/\beta_{ik} = 1 \]  \hspace{1cm} (16)

\[ \sum_{j=1}^{m} \lambda_{jk}X_{ij} = 0 \quad \forall P_{t} \quad \text{and} \quad k/\beta_{ik} = 0. \]  \hspace{1cm} (17)

Constraint (15) allows a redundancy only on the requested functions (that must be present in the finished product). By modifying the value 2 by a parameter, we could easily accept that certain functions appear more than twice in a finished product,
but this does not seem very realistic from an industrial point-of-view. We could also impose a parameter that depends on $F_k$, which means that we apply a redundancy number for each function and for each product. Constraint (16) guarantees that the needed functions have to be present at least once. Finally, constraint (17) prevents standardization.

If we wish in addition to limit the number of redundancies, we must count the total number of functions present in the product’s bill of materials and to compare it with the number of the product’s requested functions. This constraint is the following one:

$$\sum_{j=1}^{m} \sum_{i=1}^{n} \lambda_{jk} X_{ij} \leq f_i + \beta_i \quad \forall P_i, \quad (18)$$

In that case, $\beta_i$ gives the number of functions generated by the redundancy that will be tolerated.

### E. Limited Standardization With Redundancy

In the most general case, it is possible to have a redundancy and extra functions not requested in the finished product at the same time. The model then has to contain constraints (15) and (16) to allow the redundancy, constraint (13) to allow the standardization, and constraint (14) to limit the number of extra functions. In that case, the parameter $\alpha_i$ will represent the number of extra functions, including both those stemming from the redundancy and those stemming from the standardization.

To differentiate the supplementary functions according to their origin, the addition of following two constraints would be necessary

$$\sum_{k/b_k=1}^{m} \sum_{j=1}^{n} \lambda_{jk} X_{ij} \leq f_i + \beta_i \quad \forall P_i \quad (19)$$

$$\sum_{k/b_k=0}^{m} \sum_{j=1}^{n} \lambda_{jk} X_{ij} \leq \gamma_i \quad \forall P_i \quad (20)$$

The parameter $\beta_i$ gives the maximum allowable number of redundant functions and the parameter $\gamma_i$ gives the maximum allowable number of extra functions.

### F. Comments on the Supply Sources

Up to now, we have looked at the impact on the model when redundancy and standardization are introduced as alternative strategies for the determination of the product’s bill of materials. Variants can also appear in the logistical part of the model. It is possible to limit the number of sites where the module $M_j$ will be produced. To do this, the following two constraints must be added:

$$\sum_{l=1}^{s} Y_{jl} \leq \eta_j Y_j \quad \forall M_j \quad (21)$$

$$\sum_{l=1}^{s} Y_{jl} \geq \epsilon_j Y_j \quad \forall M_j, \quad (22)$$

Constraint (21) demands that the number of sites not exceed $\eta_j$ for the module $M_j$, to avoid too wide a distribution of suppliers. Constraint (22) calls for production at least $\epsilon_j$ sites. This latter constraint can take the value 1, which guarantees that at least one supplier is required, but also a larger value, which calls for an increase in the number of supply sources to anticipate a stock shortage.

In this last case, it is also possible to force every supplier to produce at least a certain percentage of the total demand for the module $M_j$. This guarantees that every supplier will mass produce the item, enabling them to reduce production costs. The constraint is then

$$Q_{jl} \geq \tau_{jl} \sum_{i=1}^{n} D_{iv} X_{ij} - (1 - Y_{jl})M \quad \forall M_j \quad \text{and} \quad S_l. \quad (23)$$

The parameter $\tau_{jl}$ indicates the minimum percentage of the quantity of the module $M_j$ required that has to be manufactured at the distant site $S_l$ and $M$ is a large number.

### V. Computational Experiments

#### A. Datasets, Experimental Conditions, and Indicators

The goal of this paper is not to provide a fast solution method, but to compare scenarios in order to better understand the influence of standardization and redundancy in different contexts. So, the experiments were conducted on small examples, and the optimal solution of the models calculated with a standard optimization solver (Cplex).

The objective of the experiments was to compare the various assembly strategies presented in this paper for several cost configurations and for different time windows $T$. To achieve this, small examples were randomly generated on which the set of possible modules, the finished product set, the distant facility set, the demands $D_{iv}$, the assembly operating times $t_{ij}$, and the distant facility capacities are fixed, while the costs vary.

The individual assembly operating times $t_{ij}$ are fixed to 1, so that constraint (3) results in a limitation in the number of modules for each bill of materials. This is made to simplify the analysis of the computational results. Of course, the model works with any value of $t_{ij}$.

Fixed and variable costs associated with the bills of materials ($F^A_j$ and $V^A_j$) are defined using a square root function of $q_j$ (the number of functions in module $M_j$). The assumption is that assembling a module containing $q_j$ functions is less expensive than assembling two modules containing $q_{j1}$ and $q_{j2}$ functions, respectively, such that $q_j = q_{j1} + q_{j2}$.

To explore several cost configurations, three parameters are used:

- $X$: which represents the ratio between assembly costs and production costs. Three possible values are assigned to this parameter:
  - $A$: indicating that the assembly costs are much higher than the production costs; (In the present study the ratio $X$ is then fixed close to 5).
  - $B$: indicating that the assembly and production costs are almost equivalent; ($X$ is close to 1).
  - $C$: indicating that the production costs are higher than the assembly costs; ($X$ is less than 0.1).
- $Y$: which represents the ratio between fixed assembly costs and variable assembly costs. Three possible values are also assigned to this parameter:
  - $+$: indicating that the fixed costs are higher than the variable costs; (the ratio $Y$ is greater than 2).
— 1: indicating that the fixed and variable costs are almost equivalent; (the ratio \( Y \) is close to 1).
— —: indicating that the variable costs are higher than the fixed costs; (the ratio \( Y \) is less than 0.5).
• \( Z \): which represents the ratio between production fixed costs and production variable costs. This parameter takes the same values as \( Y \).

With these three parameters, 27 cost scenarios were generated and used in the tests. Table I describes the parameter values for each cost scenario. Each cost scenario is characterized by a specific ratio between the various problem costs. For more detail on the cost generation procedure, readers can refer to [10].

For each of the 27 scenarios, 10 instances were generated. The problem data were fixed as follows: the number of functions \( q = 8 \), the number of finished products \( n = 30 \), where each product has at least \( \eta_{\text{min}} = 3 \) functions and at most \( \eta_{\text{max}} = 6 \) functions, \( n_T = 255 \) (all possible combinations of modules) and the number of production facilities \( s = 2 \). \( T \) varied from 3 to 6. For \( T > 6 \) (\( q_{\text{max}} \)) the solution is the same as for \( T = 6 \). For \( T \leq 2 \), the final assembly will consider a maximum of two assembly operations for each final product, which does not seem reasonable from a practical point-of-view.

Table II shows the parameters \( X \), \( Y \), and \( Z \) (after resolution of the model without standardization and redundancy) for some cost scenarios with \( T = 4 \) in order to give an idea on the magnitude of these parameters.

We call the initial model without standardization and redundancy the basic model. The basic model results will always be used as the reference value. The objective functions are compared in percentage. Then, the gap between objective values is the relative deviation, calculated as the difference between the objective function of the basic model and the objective function of the alternative one, divided by the objective function of the basic model. This means that when the gap is equal to 40% then the alternative strategy reduces the basic model objective function by 40% of its value.

The tests were conducted in C++ with the Ilog Cplex 9.0 library. They were solved on a 1.6 Ghz DELL workstation with 512 MB of RAM. The results are discussed in the following sections and all the points in the figures represent the mean values on the ten instances used in the tests.

\textbf{B. Comparison of the Different Strategies}

We first analyze the profits offered by each assembly strategy in comparison with that of the basic model assembly strategy according to the various cost structures. Furthermore we assume that the demand \( D_i \) for a product \( P_i \) is a decreasing function of the number of functions in the product, when a finished product contains more options, the demand for it becomes less than if it had fewer functions. Another demand shapes will be tested in the following section.

We use the following notations:

\( |M'| \): the number of the modules selected in \( M' \) (in the following \( |M'| \) will be called the solution size).
• Module requirement: the quantity of module \( M_j \) required to assemble the finished products required:
\[
\text{Req}_j = \sum_{i=1}^{n} D_i X_{ij}.
\]
• Solution requirement: the sum of the requirements of the solution modules \( \sum_{j=1}^{n'} \sum_{i=1}^{n} D_i X_{ij} \).
• Red: designates the model with function redundancy without standardization.
• St: designates the model with partial standardization and without redundancy where \( c_k = n \forall P_i \).
• StRed: designates the model with total standardization and without redundancy.

Fig. 3 shows the gap between the objective function values of the basic model and the model with function redundancy \( (T \) is fixed to 4). We see here that the function redundancy strategy is not profitable, but only if production costs are high relative to assembly costs. Indeed, in the B zone, production costs become almost equivalent to assembly costs, and in this case we find a small gap which reaches the maximum value in the C zone, when the production costs are the highest. However, the gap is not great and does not exceed 10%.

The standardization strategy is much more profitable than the function redundancy strategy (see Figs. 4 and 5), mainly because the first strategy makes it possible to reduce the solution size (the total number of selected modules) since the possibility of finding shared modules is greater than in the basic model strategy (see Figs. 6 and 7).

With the standardization strategy, a module may be on the product’s bill of materials even though it contains more functions than the product itself. The flexibility of this strategy leads

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
Costs & C1 & C2 & C3 & C4 & C5 & C6 & C7 & C8 & C9 \\
\hline
Y & + & + & + & 1 & 1 & 1 & - & - & - \\
Z & + & 1 & - & - & 1 & - & + & 1 & - \\
\hline
\hline
X & B & B & B & B & B & B & B & B & B \\
Y & + & + & + & 1 & 1 & 1 & - & - & - \\
Z & + & 1 & - & - & 1 & - & + & 1 & - \\
\hline
\end{tabular}
\caption{TABLE I 
\textbf{COST SCENARIOS}}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
Cost Scenarios & C1 & C5 & C9 & C12 & C15 \\
\hline
Instance number & 1 & 3 & 6 & 9 & 10 \\
\hline
Total Assembly Costs & 14398,4 & 3030 & 1168 & 4004 & 1208 \\
\hline
Total Production Costs & 1335,3 & 502,3 & 208,9 & 2262 & 66317,6 \\
\hline
X & 10,8 & 6,0 & 5,6 & 1,8 & 0,2 \\
\hline
Total Fixed Assembly Costs & 13404 & 1615 & 342 & 3100 & 392 \\
\hline
Total Variable Assembly Costs & 994,4 & 1415 & 826 & 904 & 816 \\
\hline
Y & 13,5 & 1,1 & 0,4 & 3,4 & 0,5 \\
\hline
Total Fixed Production Costs & 987,8 & 274,5 & 34,6 & 356 & 43632 \\
\hline
Total Variable Production Costs & 347,5 & 227,8 & 174,3 & 1906 & 22685,6 \\
\hline
Z & 2,8 & 1,2 & 0,2 & 0,2 & 1,9 \\
\hline
\end{tabular}
\caption{TABLE II 
\textbf{PARAMETER’S VALUES FOR SOME COST SCENARIOS WITH } T = 4
The majority of products could be assembled from two modules in the standardization model optimal solution, and also some products may contain one module in their bills of materials. This leads to a solution where the total number of requirements of modules is lower (see Fig. 8), which in turn leads to a reduction in the total variable costs (see Figs. 6 and 7).

Suppose that we have a product $P$ with a demand $D$, the BOM of $P$ is composed from modules $A$, $B$ and $C$ (in the basic model without standardization). The assembly variable costs of $P$ is equal to $D_i(V_A + V_B + V_C)$ where $V_X$ is the assembly variable cost of module $X$.

With the standardization model, it is certain that $P$ will be assembled from a lower number of modules, because extra features are allowed. Suppose that with standardization, the new BOM of $P$ is composed from modules $M$ and $N$. It is obvious that $M$ and $N$ contain more functions than $A$, $B$ and $C$. The new assembly variable cost of $P$ is equal to $D_i(V_M + V_N)$. Even though $V_M$ and $V_N$ are higher than $V_A$, $V_B$ and $V_C$ (because they contain more functions) but $D_i(V_M + V_N)$ is less than $D_i(V_A + V_B + V_C)$ according to our mathematical assumptions. This is confirmed by our results.

This reasoning is valid also for the production variable cost (the manufacturing of modules $M$ and $N$ is less costly than the manufacturing of $A$, $B$ and $C$). So globally, the standardization permits to reduce the total variable costs because the total requirements of modules are reduced. In the first case, the BOM of $P$ is composed from three modules, then the total module requirement is $3.D$, whereas it is equal to $2.D$ in the second case because we use two modules.

Fig. 9 shows the objective function gap for the total standardization model. Since the points represent the mean values of the ten used instances, vertical bars are added to show the value range of each point. Some ranges are very tight like for costs 20 and 25, some others are moderately tight like for costs 2 and 10, and some others are relatively stretched like for costs 9 and 27. However, for all cost scenarios, the ranges are quite reasonable, and they indicate that there is not a great dispersion in the numerical results.

Of course, when $\alpha$ (which is the number of supplementary functions authorized per product) increases, the gap also increases (see Fig. 10) with the larger number of additional functions tolerated, as it becomes easier to reduce the solution size and the solution requirements, which in turn leads to a reduction in both the fixed and variable costs.

However, for a fixed value of $\alpha$, the gap rate decreases when $T$ increases (see Fig. 11). As explained before, the standardization strategy leads to a reduced number of modules used to
assemble a product. Thus, constraint (3) becomes less of an influence on the solution, which is why increasing $T$ does not participate significantly in the improvement of the objective function of the standardization strategy model, especially when $\alpha$ increases (see Fig. 12). At the same time, $T$ is highly important for the basic model, and its rise permits a relatively large improvement. So, increasing $T$ causes a significant fall in the objective function for the basic model, and a nonsignificant one for the standardization model. This is why the gap decreases when $T$ increases.

Of course, the maximum profit of the standardization strategy is reached with the total standardization model. It is obvious that with such a strategy the module (11111111) can be included in the bill of materials of any product, because it contains all the functions. However, the optimal solution is not always to manufacture this module. For some costs, we have to produce other modules as well (which certainly contain many functions) in order to optimize variable costs. The assembly costs of module (11111111) are very high, because it contains the whole set of functions (based on our assumptions). For some costs, it is of greater interest to produce other modules like (10111111) (which has an assembly cost that is less than that of module (11111111) because it contains fewer functions) and use it to assemble compatible products. Then, if we assemble a product $P$ from the module (10111111) instead of (11111111), we gain the following assembly variable costs: $D_i \times (CV_{111111}^A - CV_{101111}^A)$. This is the case for some costs 9, 17, 18, 21, 22, and 27, where the configuration is such that variable costs are greater than fixed costs. For these costs, the optimal solution size is greater than 1 (see Fig. 7).

From the point of view of computational time, we note that resolution of the function redundancy model and of the total standardization model is very fast, generally a few minutes. In return, when we impose a partial standardization, the resolution takes much more computational time, and in fact may require more than 4 h of computation time. This is due to the introduction of the constraint (14) in the MILP. This constraint limits the number of non-necessary functions in modules and, consequently, a module cannot replace all the modules with fewer functions. The standardization with function redundancy model also takes much more resolution time, while the optimal solution is exactly the same as that for the total standardization model.

### C. Influence of the Demand Profile and Cost Ratio Analysis

The aim of this section is to analyze the influence of the demand profile and the cost ratio on the performance of the redundancy method and the standardization one. For this, four demand profiles are explored.

- The first profile is similar to the demand profile used in the tests for the previous sections. The demand $D_i$ for a product $P$ is a decreasing function of the number of functions in the product (the demand is more important for products having a small number of functions).
- The second profile assumes that a demand $D_i$ for a product $P$ is an increasing function of the number of functions in the product (the demand is more important for products having a small number of functions).
- The third profile assumes that a demand $D_i$ for a product $P$ is more important for products having a small or large number of functions instead of medium number of functions (the demand curve has a U shape according to the product function number).
- The forth profile is the opposite of the previous one. The demand $D_i$ for a product $P$ is less important for products
TABLE III
PARAMETER’S VALUES OF COST SCENARIOS FOR $T = 4$

<table>
<thead>
<tr>
<th>Cost Scenario</th>
<th>Demand Profile</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total fixed costs (assembly + production)</td>
<td>749.7</td>
<td>1704.5</td>
<td>2077.1</td>
<td>2079.2</td>
<td>2074.7</td>
</tr>
<tr>
<td></td>
<td>Total variable costs (assembly + production)</td>
<td>419.7</td>
<td>2241.7</td>
<td>23502.8</td>
<td>205576.2</td>
<td>804405.5</td>
</tr>
<tr>
<td>Ratio (Variable costs / Fixed costs)</td>
<td>0.56</td>
<td>1.32</td>
<td>11.32</td>
<td>98.87</td>
<td>387.72</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 13. Influence of the demand profile and cost ratio for $T = 4$.

having a small or large number of functions, and it is for product with a medium number of functions.

In order to study the cost ratio influence, five different cost scenarios are generated for which the total fixed costs are fixed and the total variable costs are progressively increased.

- For the first cost scenario, the fixed costs are more important than the variable costs.
- For the second cost scenario, the fixed and variable costs are almost equivalent.
- For the other cost scenarios, the variable costs are more important and the ratio increases when moving from the third to the fifth cost scenario.

As in the previous experiments, ten instances were generated for each demand profile where each instance contains 30 finished products and 255 modules and two distant sites.

Table III shows the ratio between total variable costs and total fixed costs for each cost scenario, the numbers represent the mean values on the ten instances.

Fig. 13 shows the gaps with the objective function (total costs) for the redundancy model and the total standardization model compared with the objective function of the basic model (without standardization and without redundancy) and for each cost scenario.

The third figure column shows the number of distinct modules obtained in the optimal solution (the solution size) for each model and for each cost scenario as well. The results are given for each demand profile too, and the curve points represent the mean values on the ten used instances.

The figure confirms the previous results: the standardization strategy is much more profitable that the redundancy one. It shows also that the demand profile has no influence on the curve shapes; the real influence comes from the ratio between fixed and variable costs. When this ratio is high (first cost scenario), the objective function gap rate is very high for the standardization strategy (more than 70%). As the cost ratio decreases (moving from costs scenario 1 to cost scenario 5), the gap between basic model objective function, redundancy objective function, and standardization objective function decreases. At the same time, the solution size becomes near to 30 modules which is the same number of the demanded finished products. This means when variables costs are very high the optimal solution consists to manufacture each product directly in a distant location facility in order to minimize these costs.

VI. CONCLUSION

The objective of this article was to propose general models for the resolution of problems associated with the simultaneous configuration of a product family and its logistical chain. We began by describing an existing model, where every finished product has to contain precisely the functions that are needed, and each function has to be present only once in the product’s bill of materials. We then proposed models which allow for controlled standardization and/or redundancy.

However, what is the advantage of partial standardization or function redundancy authorization? This is the question that we attempted to answer with the numerical tests presented in this paper. Indeed, the authorization of function redundancy does not seem to be a profitable strategy of interest. The expected gains do not exceed 10% in the best case. The standardization strategy, by contrast, is of much greater interest, with the potential of significantly higher profits, notwithstanding the cost configuration. The advantage of the standardization strategy is that it leads to a reduction in the solution size (thereby reducing the fixed costs) and also the number of modules used in a bill of materials, which reduces the total number of modules needed (thereby reducing the variable costs). The last tests indicate that demand’s profile does not have a significant influence on the objective function gaps. However the cost ratio has a great influence because when this ratio is increasingly high the three assembly strategies lead practically to the same solution.

We can summarize our results in a few succinct points.

- From an economic point-of-view, the standardization strategy leads to greater benefits than redundancy.
- The redundancy strategy is practically not interesting except when production costs are greater than assembly costs.
- The standardization strategy is much more profitable mainly when fixed costs are greater than variable costs.
- Total standardization leads generally to offer very few different products except when variable costs are greater than fixed costs.
• When variables costs become such higher that we can ignore fixed costs, then the basic model, the standardization and the redundancy strategies, provides the same optimal solution.

Our study is limited by the following assumptions: we supposed that all modules can be manufactured and the assembly costs of modules can be represented as a square root function on the number of functions of modules. Therefore, it would be interesting to consider other mathematical functions to model the module assembly costs and to introduce some restrictions on module feasibility.

These mathematical models are difficult to solve (in terms of complexity theory), and therefore almost impossible to solve in the case of industry-wide problems. That is why a heuristic approach has to be investigated. A previous method based on a taboo search algorithm has been developed and tested [11], and has been shown to perform well on the basic model. It can resolve instances up to sizes of 20 functions per finished product, the initial solution was greatly improved after six hours computational time. An adaptation of the taboo algorithm can be easily implemented to take into account redundancy and standardization models. It would be interesting to extend this method to the resolution of the other models.

The modular approach presented here implicitly considers the bill of materials in one level, that is, a bill of materials where the finished product is assembled directly from a set of independent modules. An interesting track would be to address the problem with bills of materials of depth greater than 1, or, in other words, bills of materials where the modules may themselves be assembled from smaller modules, with the possibility of dedicating some sites to the assembly of small modules and others to the assembly of large modules (even of finished products) from the small ones.

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